THE MULTIPLICITIES OF NON-ACYCLIC SL_2-REPRESENTATIONS AND L-FUNCTIONS OF TWISTED WHITEHEAD LINKS

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We briefly survey a joint work [2] with Léo Bénard, Ryoto Tange, and Anh T. Tran, which is a continuation of our previous work [8] (See also [6, 9, 1, 7] and [4, 5, 3]).

We consider a natural divisor on $SL_2\mathbb{C}$-character varieties of knots and links, given by the so-called acyclic Reidemeister torsion. We provide a geometric interpretation of this divisor. We focus on the particular family of odd twisted Whitehead links W_{2k-1}, where we show that this divisor has multiplicity two. Moreover, we apply these results to the study of the L-functions of the universal deformations of representations over finite fields of twisted Whitehead links.
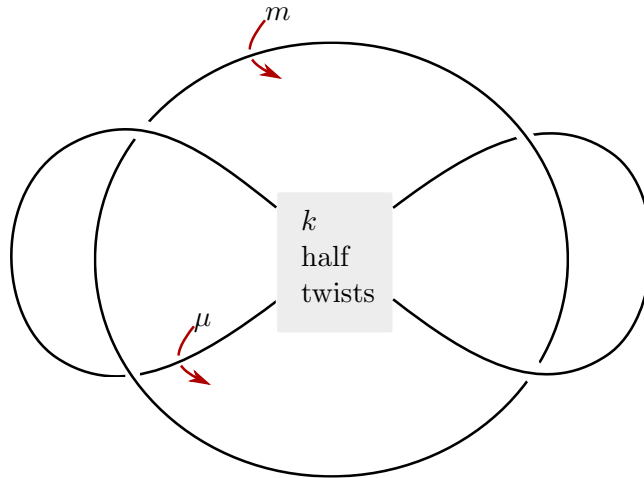

FIGURE 1. The twisted Whitehead link W_k

REFERENCES

[1] Léo Bénard. *Torsion function on character varieties*, Osaka J. Math. **58** (2021), no. 2, 291–318.

[2] Léo Bénard, Ryoto Tange, Anh T. Tran, and Jun Ueki. *Multiplicity of non-acyclic SL_2-representations and L-functions of twisted Whitehead links*, prepirnt, 2023. arXiv:2303.15941

[3] Takahiro Kitayama, Masanori Morishita, Ryoto Tange, and Yuji Terashima. *On certain L-functions for deformations of knot group representations*, Trans. Amer. Math. Soc. **370** (2018), 3171–3195.

[4] Barry Mazur. *The theme of p-adic variation, Mathematics*: frontiers and perspectives, Amer. Math. Soc., Providence, RI, 2000, 433–459.

[5] Masanori Morishita, Yu Takakura, Yuji Terashima, and Jun Ueki. *On the universal deformations for SL_2-representations of knot groups*, Tohoku Math. J. (2) **69** (2017), no. 1, 67–84.

[6] Hoang-An Nguyen and Anh T. Tran. *Twisted Alexander polynomials of twisted Whitehead links*, New York J. Math. **25** (2019), 1240–1258.

[7] Ryoto Tange. *On adjoint homological selmer modules for SL_2-augmented tautological representations of knot groups*, Springer Proceedings in Mathematics & Statistics (PROMS), to appear (2023).

[8] Ryoto Tange, Anh T. Tran, and Jun Ueki. *Non-acyclic SL_2-representations of twist knots, -3-Dehn surgeries, and L-functions*, International Mathematics Research Notices 2022 (2021), no. 15, 11690–11731.

[9] Anh T. Tran. *The A-polynomial 2-tuple of twisted Whitehead links*, Int. J. Math. **29** (2018), no. 2, 14.