ON THE CATEGORY OF REPRESENTATIONS OF A THIRD-ORDER SEMIGROUP WITHOUT UNIT AND ZERO ELEMENTS

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The classification of the semigroups of third order (in terms of Cayley tables, up to isomorphism and antiisomorphism, was first received by T. Tamura in 1953 [1]. The minimal systems of generators and the corresponding defining relations for all such semigroups were described by V. M. Bondarenko and Y. V. Zatsikha in [2].

If one considers only commutative semigroups and only those that are not neither cyclic nor cyclic with an attached unit or zero element, then there exist, up to isomorphism, the following four semigroups (in square brackets are indicated all elements, in angular a minimal system of generators, and then the defining relations; the trivial relations for unit and zero generating elements 0 and e are not written out):

(a) \((0, b, c) = \langle b, c \rangle\): \(b^2 = 0, c^2 = 0, bc = cb = 0\);
(b) \((0, b, c) = \langle b, c \rangle\): \(b^2 = b, c^2 = c, bc = cb = 0\);
(c) \((0, b, c) = \langle b, c \rangle\): \(b^2 = 0, c^2 = c, bc = cb = 0\);
(d) \((c^2, b, c) = \langle b, c \rangle\): \(b^3 = b^2, c^3 = c\)

(is a consequence of the remaining relations),
\(b^2 = c^2, bc = cb = c\).

Except for the semigroup \((a)\), all these semigroups are of finite representation type over any field \(K\), and in this case one of the forms of studying the category of representations is the description of the Auslander algebra as the algebra of endomorphisms of the direct sum of representatives of all equivalence classes of indecomposable representations. In the simple case \((b)\), the Auslander algebra was considered as an example in [3], and in the case \((c)\) it was described in [4]. Here we consider the Auslander algebra of the semigroup \((d)\) which is denoted by \(S_d\).

Theorem 1. The Auslander algebra \(\text{Aus}_K(S_d)\) of the semigroup \(S_d\) over a field \(K\) of characteristic not equal to 2 is isomorphic to the algebra of all matrices of the form

\[
X = \begin{pmatrix}
x_{11} & 0 & 0 & 0 & 0 \\
0 & x_{22} & 0 & 0 & 0 \\
0 & 0 & x_{33} & x_{34} & x_{35} \\
0 & 0 & 0 & x_{33} & 0 \\
0 & 0 & 0 & x_{54} & x_{55}
\end{pmatrix},
\]

where \(x_{ij}\) are elements of \(K\).

Theorem 2. The Auslander algebra \(\text{Aus}_K(S_d)\) of the semigroup \(S_d\) over a field \(K\) of characteristic 2 is isomorphic to the algebra of all matrices of the form

\[
X = \begin{pmatrix}
x_{11} & x_{12} & x_{13} & 0 & 0 & 0 \\
0 & x_{11} & 0 & 0 & 0 & 0 \\
0 & x_{32} & x_{33} & 0 & 0 & 0 \\
0 & 0 & 0 & x_{44} & x_{45} & x_{46} \\
0 & 0 & 0 & 0 & x_{44} & 0 \\
0 & 0 & 0 & 0 & x_{65} & x_{66}
\end{pmatrix},
\]

where \(x_{ij}\) are elements of \(K\).
These results were obtained together with Prof. V. M. Bondarenko. The first theorem was published in [5] and the second will be published in [6].

REFERENCES


