$T = D^2 \times S^1$

$D^2 = \{|z| \le 1\}$

$S^1_r = \{|z| = r\}$

$f(z, t) = |z|^2 = |(x+iy)|^2 = x^2 + y^2$

$\mathcal{F} = \{L_r = S^1_r \times S^1 \mid r \in [0, 1]\}$

$h : T \to T$ is $\mathcal{F}$-foliated if $h(L_r) = L_{r'}$

h is $\mathcal{F}$-leaf preserving $h(L_r) = L_r$

$\mathcal{D}^{lp}(\mathcal{F})$ - group of leaf-pres.

$\mathcal{D}^{fol}(\mathcal{F})$ group of foliated

$\mathcal{D}^{lp}(\mathcal{F}, \partial T) = \{h \in \mathcal{D}^{lp}(\mathcal{F}) \mid h|_{\partial T} = id\}$

$\mathcal{D}^{fol}(\mathcal{F}, \partial T) = \ldots$

T. (M - O. Khokhliyuk) These inclusions are weak homotopy equivalences

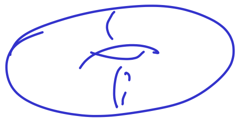
$\underline{\pi_0 \mathcal{D}(T) = A \times \mathbb{Z}_2}$ $\xrightarrow{w.h.e} \mathcal{D}^{lp}(\mathcal{F}) \xrightarrow{h.e.} \mathcal{D}^{fol}(\mathcal{F}) \hookrightarrow \mathcal{D}(T) \simeq A \times T^2$

$id_T \xrightarrow{w.h.e.} \mathcal{D}^{lp}(\mathcal{F}, \partial T) \xrightarrow{h.e} \mathcal{D}^{fol}(\mathcal{F}, \partial T) \hookrightarrow \mathcal{D}(T, \partial T) \simeq \bullet$

$id_T \hookrightarrow \mathcal{D}(T, \partial T)$ - contractible (N. Ivanov - 1970)

$A = \left\{ \begin{pmatrix} \varepsilon & n \\ 0 & \delta \end{pmatrix} \;\middle|\; \begin{matrix} \varepsilon, \delta = \pm 1 \\ n \in \mathbb{Z} \end{matrix} \right\}$

$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

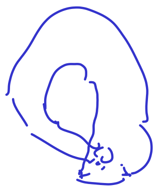


Cerf, Palais

$\mathcal{D}(T, \partial T) \to \mathcal{D}(T) \xrightarrow{p} \mathcal{D}(\partial T) = \pi_0 \mathcal{D}(\partial T) \times \underline{\mathcal{D}_{id}(\partial T)}$

$\bullet$ $\quad h \quad h|_{\partial T} \quad \simeq T^2$

K

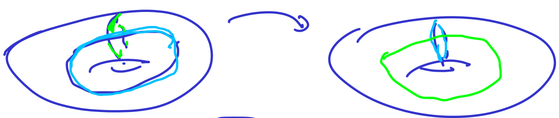


$\mathcal{D}(K, \partial K) \to \mathcal{D}(K) \to \mathcal{D}(\partial K) \simeq S^1 \times \mathbb{Z}_2 \times \mathbb{Z}_2$

$\bullet$ $\quad\quad 3 \quad\quad 2 \quad \sqcup S^1$

$\mathcal{D}($

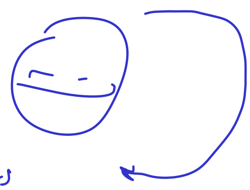
Lens spaces T_0 ξ T_1



$\xi : \partial T_0 \to \partial T_1$

$L_\xi = T_0 \cup_\xi T_1$ - 3 manifold Lens space

① $\xi = id_{\partial T}$: $L_\xi = S^2 \times S^1$



② ξ - exchanges longitude and meridians

$L_\xi = S^3 = \mathbb{R}^3 \cup \infty$

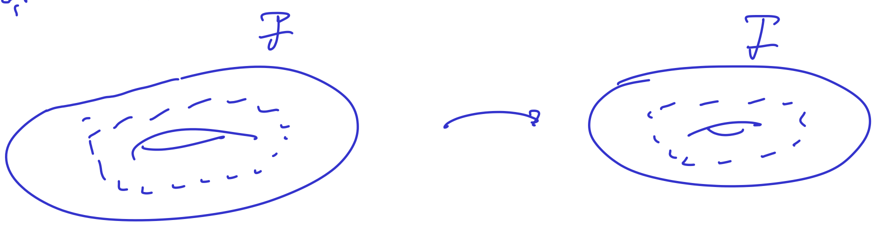
$L_\xi \simeq L_{\xi'}$ $\begin{cases} \xi \sim \xi' \text{ - isotopic} \\ g|_{\partial T} \xi h^{-1}|_{\partial T} \quad h \text{ and } g \text{ - are diffeos of } T \\ \text{exchang } T_0\ T' \quad \xi \leftrightarrow \xi^{-1} \end{cases}$

$A = \begin{pmatrix} r & p \\ s & q \end{pmatrix}$ $\quad rq - sp = -1$ $\quad L_{p,q}$

$L_{0,1} = S^2 \times S^1$ $\qquad L_{p,q}$ $\qquad 1 \le q < p$

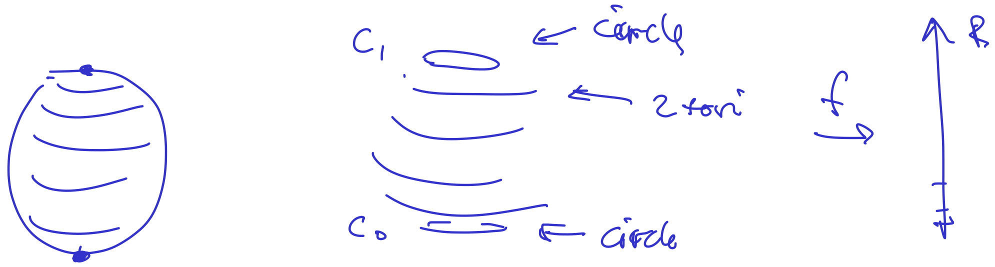
$L_{1,0} = S^3$ $\qquad p \ge 3$ $\qquad q \leftrightarrow q^{-1} \bmod p$

$L_{2,1} = \mathbb{RP}^3$



$\mathcal{F}_{pq}$ – foliation on L_{pq} with 2 singular circles and parallel 2-tori

polar



$$\mathcal{D}^{lp}(\mathcal{F}_{p,q}) \hookrightarrow \mathcal{D}^{fol}_{+}(\mathcal{F}_{p,q}) = \{h \mid h(C_0) = C_0 \quad h(C_1) = C_1\} = \mathcal{D}^{fol}(\mathcal{F}_{p,q})$$

$S^2 \times S^1 \quad S^3 \quad \mathbb{RP}^3 \qquad q^2 \neq \pm 1 \bmod p$

Th. $\mathcal{D}^{lp}(\mathcal{F}_{pq}) \overset{(*)}{\hookrightarrow} \mathcal{D}^{fol}_{+}(\mathcal{F}_{p,q})$

is strong deform. retract of

$q^2 \neq \pm 1 \bmod p \qquad p \ge 3$

Smale conj

$\mathrm{Isom}(L_{pq}) \overset{w.h.e.}{\hookrightarrow} \mathcal{D}^{lp}(\mathcal{F}_{p,q}) \overset{(*)}{\hookrightarrow} \mathcal{D}^{fol}_{+}(\mathcal{F}_{pq}) \equiv \mathcal{D}^{fol}(\mathcal{F}_{pq}) \overset{w.h.e}{\hookrightarrow} \mathcal{D}(L_{pq})$

W.h.e

elliptic metric

$\mathrm{Isom}(L_{pq}) \overset{h.e.}{\hookrightarrow} \mathrm{Diff}(L_{p,q})$

$S^3 \longrightarrow L_{p,q}$

$\mathbb{Z}_p \qquad p$

Smale Conjecture $\qquad O(4) \subset \mathcal{D}(S^3)$

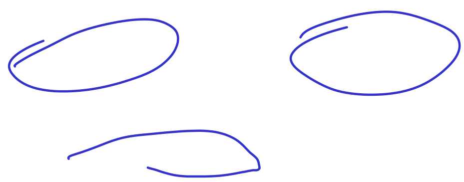
$K \underset{\partial K \to \partial K}{\sqcup} K = S^2 \tilde{\times} S^1 \qquad \mathcal{D}^{fol}(\mathcal{F}) = \underset{8}{\sqcup} S^1$

$\mathcal{D}^{lp}(\mathcal{F}) \subset \mathcal{D}^{fol}_{+}(\mathcal{F}) = \underset{4}{\sqcup} S^1 \simeq \mathcal{D}(K)$

$\mathcal{D}(\partial K)$

Fukui 1970

$\mathcal{D}(\mathcal{F}_1) = S^1$



$\mathcal{D}_{id}(\mathcal{F}) = S^1$