Multiplicative b-homogeneralized Derivations of Associative Rings

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In this manuscript, we present multiplicative b-homogeneralized derivation on an associative ring R and discuss certain differential (functional) identities having multiplicative b-homogeneralized derivation. Investigating the centralizer of suitable subset over semiprime rings that admit multiplicative b-homogeneralized derivation enhances some outcomes in the literature. We refer the reader to [4] and [2] for more details.

As is well known, the problem of linear mappings preserving fixed products is a very interesting item in the field of operator algebra. Derivations that can be completely determined by the local action on some subsets of algebra have attracted attention of many researchers. The Martindale ring of quotients of a prime ring R was introduced in [6] as a tool for studying rings satisfying a polynomial identity. The concept was extended to semiprime rings in [5]. Historically, the study of derivation was initiated during the 1950s and 1960s. Derivations of rings got a tremendous development in 1957, when [3] established two very striking results in the case of prime rings.

Named that R is a semiprime when R satisfy the expression $r_1Rr_1 = 0$ which yields $r_1 = 0$ and R is prime if $r_1Rr_2 = 0$ which supply two options there either $r_1 = 0$ or $r_2 = 0$. As a factual information about the connection between the previous concepts a prime and semiprime ring mentioned as following: A prime ring forms another kind of ring, which is a semiprime, while the converse, unfortunately, is not always true.

When a ring R admits for all $r_1, r_2 \in R$ satisfying Leibniz's rule, which is $d(r_1r_2) = d(r_1)r_2 + r_1d(r_2)$ then a derivation is that an additive map $d\colon R \to R$. Whenever for all $r_1, r_2 \in R$ there exists an identity $D(r_1r_2) = D(r_1)r_2 + r_1d(r_2)$. Then, D is an additive mapping defined as $D\colon R \to R$ is recorded as a generalized, *i.e.* a generalized derivation, where d worked as an additive mapping derivation over R.

In 2000, a classical definition of homoderivation posted in El Sofy's article [1], where he was described an additive mapping a homoderivation concerning a ring R like ψ from R to R satisfying $\psi(xy) = \psi(x)\psi(y) + \psi(x)y + x\psi(y)$ where x and y belong to R. Moreover, mapping $F\colon R \to Q_{mr}$ associated with derivation (need not be additive) $d\colon R \to R$ such that $F(\sigma\tau) = F(\sigma)\tau + b\tau d(\tau)$ holds for all $\sigma, \tau \in R$ and any fixed $0 \neq b \in Q_s \subset Q_{mr}$. If F is additive (not necessarily additive), then F is called b-generalized derivation (multiplicative b-generalized.

Definition 1. Suppose that R is an associative ring, mapping $F\colon R \to Q_{mr}$ associated with homoderivation $d\colon R \to R$ such that $F(\sigma\tau) = F(\sigma)F(\tau) + F(\sigma)\tau + b\tau d(\tau)$ holds for all $\sigma, \tau \in R$ and any fixed $0 \neq b \in Q_s \subset Q_{mr}$. When F (is not necessarily additive), then F is called b-homogeneralized derivation (multiplicative b-homogeneralized).

Theorem 2. *Let R be a semiprime ring and K be a nonzero dense ideal of R. Suppose $F\colon R \to Q_{mr}$ is a multiplicative b-homogeneralized derivation associated with derivation $d\colon R \to R$ satisfying the condition $[F(\sigma), \tau] \in Z(R)$ for all $\sigma, \tau \in K$ and any $0 \neq b \in Q_s \subseteq Q_{mr}$.*

Theorem 3. *Let R be a semiprime ring and K be a nonzero dense ideal of R. Assume $F\colon R \to Q_{mr}$ is a multiplicative b-generalized derivation associated with derivation $d\colon R \to R$ such that $F([\sigma, \tau]) = 0$ for all $\sigma, \tau \in K$ and any $0 \neq b \in Q_s \subseteq Q_{mr}$. Then either d is commuting over R is commutative or $[\sigma, b] = 0$.*

References

[1] El Sofy Aly M M.*Rings with some kinds of mappings*. M.Sc. Thesis. Cairo University. Branch of Fayoum. 2000.

[2] Mehsin Jabel Atteya, *New types of permuting n-derivations with their applications to associative rings*, *Symmetry*, 12,(2020) 46;doi:10.3390/sym12010046,www.mdpi.com/journal/symmetry.

[3] E.C. Posner, Derivations in prime rings,*Proc. Amer. Math. Soc.*8: 1093- 1100 (1957).

[4] Fošner A. and Mehsin Jabel Atteya,*Semigeneralized semiderivations of semiprime rings*, *AIP Conference Proceedings* 2037, 020010 .2018; https://doi.org/10.1063/1.5078465.

[5] S. A. Amitsur, *On rings of quotients*,*Sympos. Math.*, **8**, (1972), 149-164.

[6] W. S. Martindale, *Prime rings satisfying a generalized polynomial identity*,*J. Algebra* **12**,(1969), 576-584.