On 2-convex embeddings of non-orientable surfaces in four-dimensional Euclidean space

Dmitry V. Bolotov

(B. Verkin Institute for Low Temperature Physics and Engineering of the National Academy of Sciences of Ukraine, 47 Nauky Ave., Kharkiv, 61103, Ukraine)

E-mail: bolotov@ilt.kharkov.ua

Let us recall the definition of k-convexity of a subset of a Euclidean space (see [1]).

Definition 1. A subset $C \subset E^n$ of Euclidean space is called k-convex if through each point $x \in E^n \setminus C$ there passes a k-dimensional plane that does not intersect C.

Note that the usual convexity corresponds to case k = n - 1. We present the following result.

Theorem 2. The Projective plane and the Klein bottle do not admit a 2-convex embedding in a four-dimensional Euclidean space if the embedding is assumed to be C^2 -smooth or is a PL-embedding such that the valence of the vertices does not exceed five.

References

[1] Yu. B. Zelinsky, Vypuklost'. Izbrannyye glavy, Inst. matematiki NAN Ukrainy, Kiev, 2012.