On representation type of incident algebras of extensions of positive posets

Vitaliy Bondarenko
(Institute of Mathematics of NAS of Ukraine, Kyiv, Ukraine)
E-mail: vitalij.bond@gmail.com

Maryna Styopochkina
(Polissia National University, Zhytomyr, Ukraine)
E-mail: stmar@ukr.net

In 1972, P. Gabriel introduced a quadratic form for finite quivers, which was called by him the Tits quadratic form. He proved that a quiver Q is of finite representation type over a field k (i.e., has up to equivalence finitely many indecomposable representations) if and only if its Tits quadratic form is positive. This result laid the foundations of a new direction in the theory of algebras.

In 1974, Yu. A. Drozd showed that a (finite) poset S is of finite representation type if and only if its quadratic Tits form

$$q_S(z) = z_0^2 + \sum_{i\in S} z_i^2 + \sum_{i<j, i,j\in S} z_i z_j - z_0 \sum_{i\in S} z_i$$

is weakly positive, i.e., positive on the non-zero vectors with non-negative coordinates (representations of posets were introduced by L. A. Nazarova and A. V. Roiter in 1972). In contrast to quivers, the posets with weakly positive and with positive Tits forms do not coincide. Since the connected quivers having positive Tits quadratic form coincide with the quivers whose underlying graphs are (simply faced) Dynkin diagrams, the posets with positive Tits form are analogs of the Dynkin diagrams. Such posets, which are simply called positive, were classified by the authors in [1]. Up to isomorphism and duality, the positive posets consist of 5 series and 108 discrete ones.

Since the incidence algebras of positive posets are of finite representation type, it is natural to study representation type of their element-extensions. We consider cases when "new elements" are nodes (in the sence that they are comparable with all other elements). Here we formulate some consequences of our investigation.

A positive poset T is called serial if there exists infinite consequence $T \subset T_1 \subset T_2 \subset \ldots$ such that all posets T_i are also positive. By definition, the incidence algebra $\mathcal{I}_k(T)$ of a poset T of order n is the matrix algebra which consisits of all matrices $M = (m_{ij})$, $i, i \in T$ (over a field k) such that $m_{ij} = 0$ if $i > j$. Finally, we write $X > Y$ for subposets X, Y of a poset T if $x > y$ for any $x \in X, y \in Y$.

Theorem 1. *Let S be a positive poset and $\overline{S} = S \cup X$ be an extension of S with chained $X > S$. Then the incidence algebra $\mathcal{I}_k(\overline{S})$ is of infinite representation type if the length $l(X)$ of X is greater than 5.*

Theorem 2. *Let $\overline{S}$ be as in Theorem 1 but S is non-serial. Then the incidence algebra $\mathcal{I}_k(\overline{S})$ is of infinite representation type if*

(a) *the width of S is equal to 2 and $l(X) > 4$;*

(b) *the width of S is equal to 3 and $l(X) > 1$.*

In all statements the estimates are exact.

References

[1] V. M. Bondarenko, M. V. Styopochkina. (Min, max)-equivalence of partially ordered sets and the Tits quadratic form. *Collection of works of Inst. of Math. NAS Ukraine – Problems of Analysis and Algebra*, 2(3) : 18–58, 2005.