New Combinatorial Invariants of Doubly Periodic Tangles

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Doubly periodic tangles, or *DP tangles*, are complex entangled structures consisting of curves embedded in the thickened plane $\mathbb{E}^2 \times I$ that are periodically repeated in two directions. Hence, DP tangles can be defined as lifts of links in the thickened torus, $T^2 \times I$. The topological classification of DP tangles is at least as hard a problem as the full classification of knots and links in the three-space and is approached by constructing topological invariants. To reduce the complexity of this problem, the idea is to consider the quotient of a DP diagram under a periodic lattice, namely a link diagram in the (flat) torus T^2 that we call *(flat) motif*. This approach leads to a diagrammatic theory of the topological equivalence of DP tangles, which has been established in [1] on the level of motifs, and that generalizes works initiated by Grishanov et al. related to textiles ([4, 5]).

In this talk we will first establish the mathematical framework of the topological theory of DP tangles in order to characterize the notion of *equivalence* between DP tangles and between their flat motifs. Time permits, we shall further generalize these results to other diagrammatic categories, such as framed, virtual, singular, pseudo and bonded DP tangles, which could be used in novel applications. We will then introduce new topological invariants of DP tangles. In particular, we will present the notion of *axis-motif*, that is a set of arcs in the flat torus which can be viewed as a blueprint of a DP tangle capturing the different directions along which its components are organized. This will lead to the definition of the *directional type* of the DP tangle, which constitutes a topological invariant of DP tangles ([2]). We will also introduce the concept of *density* of a motif τ, defined in terms of the total number of arcs of the axis-motif of τ, which gives rise to a new invariant called *density of the DP tangle* τ_∞, defined as the minimal density over all axis-motifs of τ_∞. However, we will note that this topological invariant is not strong enough to distinguish two DP tangles of different directional types. Thus, by using the fact that the set of arcs of an axis-motif of a motif τ can be partitioned into a specific triple of integers, that we call *arc-triple of* τ, we will present a stronger invariant of DP tangles, called *minimal arc-triple*. This notion leads to a characterization of the directional type of a DP tangle by its minimal arc-triple. All the above invariants of DP tangles are measures that naturally inform on their topological complexity, they refer to global topological properties of theirs, and they add to the list of the existing invariants.

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References

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