Interplay of Global Implicit Functions and Critical Point Theory in Infinite Dimensional Spaces

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Consider a nonlinear equation of the form:

$$\Phi(\mathtt{e}, \mathtt{g}) = \mathbf{0}, \tag{1}$$

where $\mathtt{e}, \mathtt{g}$, and $\mathbf{0}$ belong to arbitrary Fréchet spaces, and $\mathbf{0}$ represents the zero element. We establish sufficient conditions under which it is possible to globally and uniquely solve Equation (1) for $\mathtt{g}$ in terms of $\mathtt{e}$, with the solution mapping $\mathcal{K}$ being differentiable, such that Φ does not lose the derivative.

Applying the obtained global implicit function theorem, we will establish sufficient conditions for the global existence and uniqueness of the solution over the entire time of the following initial value problem that involves the loss of one derivative:

$$y'(t) = \Phi(t, y(t), e), \tag{2}$$

where the initial conditions are fixed both in time and in arbitrary Fréchet spaces.

We also generalize the Lagrange multiplier method, which involves finding critical points of a mapping subject to a set of constraints, and apply the results to extend the Nehari method for locating critical points.

The full details can be found in [1].

References

[1] Kaveh Eftekharinasab. Global Implicit Function Theorems and Critical Point Theory in Fréchet Spaces. *arXiv preprint arXiv:2404.00286*, 2024.