Nonlinear interpolation of α-Holderian mappings with applications to quasilinear PDEs

Maria Rosaria Formica
(University of Napoli "Parthenope", ITALY)
E-mail: mara.formica@uniparthenope.it

The Marcinkiewicz interpolation theorems for linear operators acting on Lebesgue spaces turned out to be a powerful tool for studying regularity of solutions for linear PDEs in L^p-spaces. The K-method introduced by J. Peetre ([5, 6]) allowed to extend the study of regularity of solutions of linear equations on spaces different from L^p-spaces. The main difficulty to apply Peetre's definition is the identification of the interpolation spaces between two normed spaces embedded in a same topological space. In [2, 3, 4] we did such a study with applications to linear PDEs using new non-standard spaces as grand or small Lebesgue spaces and $G\Gamma$-gamma spaces.
In [7] L. Tartar gave interpolation results on nonlinear Hölderian mappings (which include Lipschitz mappings) and applied them to a variety of boundary value problems as bilinear applications, semi-linear PDEs but also on variational inequalities.
In this talk we present some results contained in the recent paper [1], were we extend Tartar's results on nonlinear interpolation of α-Hölderian mappings $\mathcal{T}$ by studying the action of the mappings $\mathcal{T}$ on K-functionals and between interpolation spaces with logarithm functors. Therefore, we identify some interpolation spaces using couples of Lebesgue or Lorentz spaces, recovering spaces as Lorentz–Zygmund spaces or $G\Gamma$-gamma spaces.

We apply these results to obtain regularity on the gradient of the weak or entropic-renormalized solution u to quasilinear equations of the form

$$-\mathrm{div}(\widehat{a}(\nabla u)) + V(x;u) = f, \qquad u = 0 \text{ on } \partial\Omega, \tag{1}$$

associated to the Dirichlet homogeneous condition on the boundary, where Ω is a bounded smooth domain of $\mathbb{R}^n$, $\widehat{a}(\nabla u) = |\nabla u|^{p-2}\nabla u$, V is a nonlinear potential and f belongs to non-standard spaces like Lorentz-Zygmund spaces. We also show that the mapping $\mathcal{T}: \ \mathcal{T}f = \nabla u$ is locally or globally α-Hölderian under suitable values of α and appropriate assumptions on V and $\widehat{a}$.

Furthermore, also the anisotropic version or the variable exponents version of the Laplacian are considered.

References

[1] Ahmed I., Fiorenza A., Formica M.R., Gogatishvili A., El Hamidi A., Rakotoson J.M., Quasilinear PDES, Interpolation spaces and Hölderian mappings, *Analysis Math.* **49** (4) (2023), 895-950.

[2] Ahmed I., Fiorenza A., Formica M.R., Gogatishvili A., Rakotoson J.M., Some new results related to Lorentz GΓ spaces and interpolation, *J. Math. Anal. Appl.* **483** (2020), 123623.

[3] Fiorenza A., Formica M.R, Gogatishvili A., Kopaliani K., Rakotoson J.M., Characterization of interpolation between grand, small or classical Lebesgue spaces, *Nonlinear Anal.* **177** (2018), 422-453.

[4] Fiorenza A., Formica M.R., Rakotoson J.M., Pointwise estimates for GΓ-functions and applications, *Differential Integral Equations* **30** (2017), 809-824.

[5] Peetre J., A theory of interpolation of normed spaces, Notas de Matematica, **39** (1968), 1-86.

[6] Peetre J., A new approach in interpolation spaces, *Studia Math.* **34** (1970), 23-42.

[7] Tartar L., Interpolation non linéaire et régularité, *J. Funct. Anal.*, **9** (1972), 469-489.