On the multiplicative order convergence on Banach lattice f-algebras

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Let E ,F be Archimedean vector lattices.The Fremlin tensor product $E\overline{\bigotimes}F$ of E and F was introduced by Fremlin in [6]. $E\overline{\bigotimes}F$ contains the algebraic tensor product $E\bigotimes F$ as an ordered vector subspace satisfying density properties. The Fremlin projective tensor product $E\widehat{\bigotimes}F$ of Banach lattices E and F is a Banach lattice, [7]. It contains the Fremlin tensor product $E\overline{\bigotimes}F$ as a norm dense normed lattice. It is known that the Fremlin tensor product $A\overline{\bigotimes}B$ is an f-algebra if A and B are f-algebras, [4,5]. Also, we know that if A and B are Banach lattice f-algebras, then the Fremlin projective tensor product$A\widehat{\bigotimes}B$ of A and B is a Banach lattice f-algebra, [9].

A vector lattice E under an associative multiplication is called a lattice ordered algebra whenever the multiplication makes E an algebra with the usual properties and multiplication of positive elements in E is positive. A lattice ordered algebra A is called an f-algebra if $x \wedge y = 0$ for every $x, y \in A$ implies $(zx) \wedge y = (xz) \wedge y = 0$ for all $z \in A^+$, where A^+ denotes the positive part of A. . A Banach algebra A is called a Banach lattice algebra if A is a Banach lattice and the multiplication of positive elements in A is positive. A Banach lattice algebra A is called a Banach lattice f-algebra if A is an f-algebra.

Definition 1. A net (x_α) in an Archimedean vector lattice E is called order convergent to $x \in E$ if there exists a net (y_β) satisfying $y_\beta \downarrow 0$, and for any β there exists α_β such that $|x_\alpha - x| \leq y_\beta$ for all $\alpha \geq \alpha_\beta$.

Definition 2. Let A be an f-algebra. A net (x_α) in A is called multiplicative order convergent to $x \in A$ if $|x_\alpha - x|.u \to 0$ convergences in order for all $u \in A^+$.

Definition 3. A lattice ordered algebra A is called a normed lattice ordered algebra whenever it is a normed vector lattice and $\|x.y\| \leq \|x\|\|y\|$ holds for all $x, y \in A$.

Definition 4. A net (x_α) in a normed lattice ordered algebra A is called multiplicative norm convergent to $x \in A$ if $\||x_\alpha - x|.u\| \to 0$ for all $u \in A^+$.

The concept of convergence in f-algebras related to multiplication was given before by A. Aydin, [1,2]. The studies under the unbounded order convergence and unbounded norm convergence in vector lattices and Banach lattices were done before by many authors.

O. Zabeti applied the unbounded order convergence to the Fremlin tensor product of vector lattices and the unbounded norm convergence to the Fremlin projective tensor product of Banach lattices in [10].

Our aim is to investigate the multiplicative order convergence in the Fremlin tensor product of two f-algebras and the multiplicative norm convergence in the Fremlin projective tensor product of two Banach lattice f-algebras.

For this subject, we give the following references.

References

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