The some solution of the Bryan-Pidduck equation

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The nonlinear integrodifferential Boltzmann equation [1] that describes the evolution of rarefied gases is one of the main equations of the kinetic theory of gases. The Boltzmann equation for the model of rough spheres (or the Bryan–Pidduck equation) has the form

$$D(f) = Q(f,f); \tag{1}$$

$$D(f) \equiv \frac{\partial f}{\partial t} + \left(V, \frac{\partial f}{\partial x}\right), \tag{2}$$

$$\begin{gathered} Q(f,f) \equiv \frac{d^2}{2} \int_{R^3} dV_1 \int_{R^3} d\omega_1 \int_{\Sigma} d\alpha B(V - V_1, \alpha) \\ \Big[f(t, V_1^*, x, \omega_1^*) f(t, V^*, x, \omega^*) - f(t, V, x, \omega) f(t, V_1, x, \omega_1)\Big]. \end{gathered} \tag{3}$$

The problem of determination of the exact and approximate solutions of the Bryan–Pidduck equation in the explicit form is quite urgent. At present, the sole known exact solution of the Boltzmann equation is an expression usually called the Maxwell distribution or simply Maxwellian (after J. C. Maxwell, Scottish physicist). In the case of Maxwellians M, we get

$$D(f) = 0, \quad Q(f,f) = 0. \tag{4}$$

The solution to this equation (1)-(3) will be look for in the next form

$$f(t, x, V, \omega, u) = \int_{\mathbb{R}^3} \varphi(t, x, u) M(V, \omega, u) du. \tag{5}$$

As a measure of the deviation between the parts of equation (1) we will consider a uniform-integral error of the form:

$$\Delta = \sup_{(t,x)\in\mathbb{R}^4} \int_{\mathbb{R}^3} dV \int_{\mathbb{R}^3} d\omega \Big| D(f) - Q(f,f) \Big|. \tag{6}$$

In the paper [2], we were obtained sufficient conditions for the coefficient functions and hydrodynamic parameters appearing in the distribution, which enable one to make the analyzed error (6) as small as desired.

References

[1] S. Chapman and T.G. Cowling. The Mathematical Theory of Non-Uniform Gases. *Cambridge Univ. Press,* Cambridge, 1952.

[2] V.D. Gordevskyy and O.O. Hukalov. *Continual distribution for the Bryan-Pidduck Equation*, *Ukrainian Mathematical Journal.* Vol. 72, No. 11, 1715–1723.