About Rolewicz theorem on inversion of continuous bijection between F-spaces

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Well-known result of Stefan Banach states that if X and Y are F-spaces and $f : X \to Y$ is a bijective additive continuous mapping, then the inverse mapping $f^{-1} : Y \to X$ is continuous. In general case the inverse mapping can be everywhere discontinuous.

In article [1] Stefan Rolewicz presented sufficient conditions on spaces X and Y under which the inverse mapping to a continuous bijection belongs to the first Baire class.

Theorem 1 (Rolewicz, 1958)**.** *Let X, Y be F-spaces and let X be separable locally compact. Then for every continuous bijection $f : X \to Y$ the inverse mapping $f^{-1} : Y \to X$ is Baire 1.*

The aim of this talk is a discussion of possible generalizations of the above mentioned result of Rolewicz on spaces X which are not linear. In order to do this we introduce a notion of weak Rolewicz space and prove the auxiliary fact about uniform limit of Baire 1 functions which is of self contained interest and extends corresponding results from [2].

Definition 2. A metric space (X, d) is called a *weak Rolewicz space*, if there exist $C > 0$, a sequence $(\varepsilon_n)_{n=1}^{\infty}$ of positive reals which tends to zero and a sequence $(R_n)_{n=1}^{\infty}$ of functions $R_n : X \times X \to X$ such that for all $x, y \in X$

(1) if $d(x, y) \le \varepsilon_n$, then $R_n(x, y) = x$,
(2) $d(R_n(x, y), y) \le C \cdot \varepsilon_n$ for $n = 1, 2, \ldots$.

Every convex subset of a metric vector space is an example of a weak Rolewicz space. Moreover, there are zero dimensional examples of Rolewicz spaces.

Proposition 3. *If Y is a weak Rolewicz space, then a uniform limit $f : X \to Y$ of a sequence of Baire 1 functions $f_n : X \to Y$ belongs to the first Baire class.*

The next theorem is the main result of the talk.

Theorem 4. *Let X, Y be metric spaces and X is locally compact weak Rolewicz space. Then for every continuous bijection $f : X \to Y$ the inverse mapping $f^{-1} : Y \to X$ is Baire 1.*

References

[1] Stefan Rolewicz. On inversion of non-linear transformations, *Studia Mathematica*, 17 : 79–83, 1958.
[2] Olena Karlova, Mykhaylo Lukan, R-spaces and uniform limit of sequences of the first Baire class, *Bukovinian Mathematical Journal*, 7 (2) : 39–47, 2019.