Integral problem for system of partial differential equations of third order

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Let $H(\mathbb{R}_+ \times \mathbb{R}^n)$ be a class of entire functions on $\mathbb{R}$, K_L is a class of quasipolynomials of the form $\varphi(x) = \sum_{i=1}^{n} Q_r(x) \exp[\alpha_r x]$, where $\alpha_r \in L \subseteq \mathbb{C}$, $\alpha_k \neq \alpha_l$, for $k \neq l$, $Q_r(x)$ are given polynomials.

Each quasipolynomial defines a differential operator $\varphi\left(\frac{\partial}{\partial\lambda}\right)$ of finite order on the class of entire function, in the form $\sum_{r=1}^{m} Q_r\left(\frac{\partial}{\partial\lambda}\right) \exp\left[\alpha_i \frac{\partial}{\partial\lambda}\right]\Bigg|_{\lambda=0}$.

In the strip $\Omega = \{(t,x) \in \mathbb{R}^{n+1} : t \in \{([T_1,T_2] \cup [T_3,T_4]), x \in \mathbb{R}^n\}$, we consider of the system of equations

$$\frac{\partial^3 U_i}{\partial t^3} + \sum_{j=1}^{n} \left\{ a_{ij}\left(\frac{\partial}{\partial x}\right) \frac{\partial^2 U_j}{\partial t^2} + b_{ij}\left(\frac{\partial}{\partial x}\right) \frac{\partial U_j}{\partial t} + c_{ij}\left(\frac{\partial}{\partial x}\right) \right\} U_j(t,x) = 0, \tag{1}$$

$$\int_{T_1}^{T_2} U_{ik}(t,x)dt + \int_{T_3}^{T_4} U_{ik}(t,x)dt = \varphi_{ik}(x), \quad k = 1,2,3, \tag{2}$$

$$\int_{T_1}^{T_2} tU_{ik}(t,x)dt + \int_{T_3}^{T_4} tU_{ik}(t,x)dt = \varphi_{ik}(x). \quad i = 1,...,n, \tag{3}$$

$$\int_{T_1}^{T_2} t^2 U_{ik}(t,x)dt + \int_{T_3}^{T_4} t^2 U_{ik}(t,x)dt = \varphi_{ik}(x). \tag{4}$$

Where $a_{ij}\left(\frac{\partial}{\partial x}\right), b_{ij}\left(\frac{\partial}{\partial x}\right), c_{ij}\left(\frac{\partial}{\partial x}\right)$, are differential expression with entire symbols $a_{ij}(\lambda) \neq 0, b_{ij}(\lambda) \neq 0, c_{ij}(\lambda) \neq 0$.

Let be $\eta(\lambda) = \int_{T_1}^{T_2} W^{n-1}(t,\lambda)dt + \int_{T_3}^{T_4} W^{n-1}(t,\lambda)dt$ is a certain function $W(t,\lambda)$ is a solution of equation $\left(\frac{d^n}{dt^n} + \sum_{i=1}^{n} a_i(\lambda) \frac{d^{n-i}}{dt^{n-i}}\right) W(t,\lambda) = 0$, satisfies conditions $W^n(t,\lambda)\Big|_{t=0} = 1, \quad W^{n-1}(t,\lambda)\Big|_{t=0} = 0, \quad W(t,\lambda)\Big|_{t=0} = 0$.

Denote be $P = \Big\{\Delta(\lambda) = 0, \lambda \in \mathbb{C}\Big\}$ set zeros of function $\eta(\lambda)$.

Theorem 1. ***Theorem.*** *Let $\varphi_{ik}(x) \in K_L$, $i = 1,...,n$, $j = 1,...,n$ then the class $K_{L \setminus P}$ exist and unique solution of the problem (1)-(4). Solution of the problem (1)-(4) can be represented in the form*

$$U_i(t,x) = \sum_{k=1}^{3} \sum_{p=1}^{n} \varphi_{kp}\left(\frac{\partial}{\partial x}\right) \left\{ \frac{1}{\eta(\lambda)} T_{kjp}(t,\lambda) W(t,\lambda) \exp[\lambda x] \right\} \Bigg|_{\lambda=0},$$

where $T_{kjp}(t,\lambda) = l^T\left(\frac{d}{dt},\lambda\right)$ is transpose of a matrix $\left(\frac{d}{dt},\lambda\right)$.

Solution of the problem (1) - (4) according to the differential-symbol method [1], [2] exists and unique in the class of quasi-polynomials. Be means of the differential-symbol method [1], [2] we construct of the problem (1)-(4). This problem is a continuos works [3] - [6].

References

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