Invariant transformation of generalized-recurrent-parabolic spaces that are in a quasi-geodesic mapping

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We study diffeomorphisms of pseudo-Riemannian spaces that belong to the intersection of classes of quasi-geodesic mappings (QGM) [2] with the reciprocity condition and almost-geodesic mappings of the second type [1]. We mean that QGM $f : (V_n, g_{ij}, F_i^h) \longrightarrow (\overline{V}_n, \overline{g}_{ij}, F_i^h)$ satisfies the reciprocity condition if the reverse mapping f^{-1} is also QGM.

The fundamental equations of such a mapping f in the common coordinate system (x^i) with respect to the mapping f has the form:

$$\overline{\Gamma}_{ij}^h(x) = \Gamma_{ij}^h(x) + \psi_{(i}(x)\delta_{j)}^h + \phi_{(i}(x)F_{j)}^h(x), \tag{1}$$

$$F_i^h(x) = \overline{F}_i^h(x),$$

$$g_{i\alpha}F_j^\alpha = -g_{j\alpha}F_i^\alpha, \qquad \overline{g}_{i\alpha}F_j^\alpha = -\overline{g}_{j\alpha}F_i^\alpha, \tag{2}$$

$$F_{(i,j)}^h = q_{(i}F_{j)}^h, \tag{3}$$

$$F_\alpha^h F_i^\alpha = e\delta_i^h, \qquad e = 0, \pm 1, \tag{4}$$

$$i, h, j, ... = 1, 2, ...n,$$

where $\Gamma_{ij}^h, \overline{\Gamma}_{ij}^h$ are the Christoffel symbols of $V_n, \overline{V}_n$, respectively; $\psi_i(x)$, $\phi_i(x)$, $q_i(x)$, $p_i(x)$ are certain covectors; $F_i^h(x)$ is affinor; brackets (i, j) denote the symmetrization with respect to the corresponding indices; comma «,» is a sign of the covariant derivative in respect to the connection of V_n.

We call an affinor structure F_i^h that satisfies conditions (3) a *generalized-recurrent structure* (of elliptic, hyperbolic, or parabolic type). Let us study the case of a parabolic structure ($e = 0$).

The following holds:

Theorem 1. *If there is a non-trivial QGM of generalized-recurrent-parabolic spaces $f : (V_n, g_{ij}, F_i^h) \longrightarrow (\overline{V}_n, \overline{g}_{ij}, F_i^h)$, which corresponds to the affinor F_i^h and the vector ϕ_i, then it generates another non-trivial QGM of other generalized-recurrent-parabolic spaces*

$$f_1 : (\overset{1}{V}_n, \overset{1}{g}_{ij}, \overset{1}{F}{}_i^h) \longrightarrow (\overset{1}{\overline{V}}_n, \overset{1}{\overline{g}}_{ij}, \overset{1}{F}{}_i^h),$$

which corresponds to the affinor $\overset{1}{F}{}_i^h$ and the vector $\overset{1}{\phi}_i$ and preserves the generalized recurrence vector q_i. The tensors $\overset{1}{g}_{ij}$, $\overset{1}{\overline{g}}_{ij}$, $\overset{1}{\phi}_i$, $\overset{1}{F}{}_i^h$ are given by the formulas

$$\overset{1}{g}_{ij} = e^{2\psi}\overline{g}^{\alpha\beta}g_{\alpha i}g_{\beta j},$$
$$\overset{1}{\overline{g}}_{ij} = e^{2\psi}g_{ij},$$
$$\overset{1}{g}_{ij,k} = -\overset{1}{\phi}_{\overline{i}}g_{jk} - \overset{1}{\phi}_{\overline{j}}g_{ik} - \overset{1}{\phi}_{i}F_{jk} - \overset{1}{\phi}_{j}F_{ik},$$
$$\overset{1}{F}{}^h_i \overset{def}{=} F^h_\alpha \tilde{B}^\alpha_i = F^\alpha_i \tilde{B}^h_\alpha,$$
$$B^h_i = e^{2\psi}\overline{g}^{h\alpha}g_{\alpha i}, \qquad \tilde{B}^h_i = e^{-2\psi}g^{h\alpha}\overline{g}_{\alpha i}.$$

Theorem 2. *If there is a non-trivial QGM of generalized-recurrent-parabolic spaces $f : (V_n, g_{ij}, F^h_i) \longrightarrow (\overline{V}_n, \overline{g}_{ij}, F^h_i)$, which corresponds to the affinor F^h_i and the vector ϕ_i, then it generates an infinite sequence of non-trivial QGM of other generalized-recurrent-parabolic spaces*

$$(\overset{1}{V}_n, \overset{1}{g}_{ij}, \overset{1}{F}{}^h_i) \xrightarrow{f_1} (\overset{1}{\overline{V}}_n, \overset{1}{\overline{g}}_{ij}, \overset{1}{F}{}^h_i),$$
$$\downarrow$$
$$(\overset{2}{V}_n, \overset{2}{g}_{ij}, \overset{2}{F}{}^h_i) \xrightarrow{f_2} (\overset{2}{\overline{V}}_n, \overset{2}{\overline{g}}_{ij}, \overset{2}{F}{}^h_i),$$
$$\downarrow$$
$$\cdots\cdots\cdots\cdots\cdots\cdots$$
$$\downarrow$$
$$(\overset{s}{V}_n, \overset{s}{g}_{ij}, \overset{s}{F}{}^h_i) \xrightarrow{f_s} (\overset{s}{\overline{V}}_n, \overset{s}{\overline{g}}_{ij}, \overset{s}{F}{}^h_i),$$
$$\downarrow$$
$$\cdots\cdots\cdots\cdots\cdots\cdots$$

which correspond to the affinor $\overset{s}{F}{}^h_i$ and the vector $\overset{s}{\phi}_i$ and preserve the generalized recurrence vector q_i. The tensors $\overset{s}{g}_{ij}$, $\overset{s}{\overline{g}}_{ij}$, $\overset{s}{\phi}_i$, $\overset{s}{F}{}^h_i$, are given by the formulas

$$\overset{s}{g}_{ij} = \overset{(s)}{B}{}^\alpha_i g_{\alpha j}, \qquad \overset{s}{\overline{g}}_{ij} = e^{2\psi}\overset{(s-1)}{B}{}^\alpha_i g_{\alpha j}.$$
$$\overset{s}{\phi}_i = \phi_\alpha \overset{(s)}{B}{}^\alpha_i, \qquad \overset{s}{F}{}^h_i = F^h_\alpha \overset{(s)}{\tilde{B}}{}^\alpha_i,$$

where $\overset{(s)}{B}{}^h_i$ is the s-th degree of the affinor B^h_i and we mean $\overset{(0)}{B}{}^h_i = \delta^h_i$.

References

[1] Sinyukov N.S. *Geodesic mappings of Riemannian spaces.*. M.: Nauka, Moscow, 1979.

[2] Petrov A .Z. Modeling of the paths of test particles in gravitation theory *Gravit. and the Theory of Relativity*, 4-5 : 7–21, 1968.