Persistent interaction topology in data analysis

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Abstract. Topological data analysis, as a tool for extracting topological features and characterizing geometric shapes, has had tremendous success across diverse fields. Its key mathematical techniques include persistent homology and the recently developed persistent Laplacians. However, classic mathematical models like simplicial complexes often struggle to provide a localized topological description for interactions or individual elements within a complex system involving a specific set of elements. In this work, we introduce persistent interaction homology and persistent interaction Laplacian that emphasize individual interacting elements in the system. We demonstrate the stability of persistent interaction homology as a persistent module. Furthermore, for a finite discrete set of points in the Euclidean space, we provide the construction of persistent interaction Vietoris-Rips complexes and compute their interaction homology and interaction Laplacians. The proposed methods hold significant promise for analyzing heterogeneously interactive data and emphasizing specific elements in data. Their utility for data science is demonstrated with applications to molecules.