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In this paper we construct indecomposable vector bundles associated to monads on multiprojective spaces. Specifically we establish the existence of monads on $\mathbf{P}^{2n+1} \times \mathbf{P}^{2n+1} \times \dots \times \mathbf{P}^{2n+1}$ and on $\mathbf{P}^{a_1} \times \dots \times \mathbf{P}^{a_n}$. We prove stability of the kernel bundle which is a dual of a generalized Schwarzenberger bundle associated to the monads on $X = \mathbf{P}^{2n+1} \times \mathbf{P}^{2n+1} \times \dots \times \mathbf{P}^{2n+1}$ and prove that the cohomology vector bundle which is simple, a generalization of special instanton bundles. We also prove stability of the kernel bundle and that the cohomology vector bundle associated to the monad on $\mathbf{P}^{a_1} \times \dots \times \mathbf{P}^{a_n}$ is simple. Lastly, we construct the morphisms that establish the existence of monads on $\mathbf{P}^1 \times \dots \times \mathbf{P}^1$.

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