Construction and application of quasicrystals

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We discuss three methods that generate n-dimensional quasicryatals and propose two applications of quasicrystals to data processing. The first application is to use the mapping between the physical and internal spaces of a quasi-crystal to evenly distribute data that is lost in the process of transmitting or storing information. At the same time, it is possible to unambiguously restore the rest of the data. The second application consists in the construction of special quasi-crystals that satisfy the requirements of keys of any length for the classical Vernam cipher method. Several examples of construction of quasicrystals with predetermined properties and examples of image processing that makes the loss of its part uniformly distributed are given.

Different physical phenomena arising from the interaction of incommensurate frequencies display the features of almost periodicity. A typical example is a potential field of a physical quasicrystal. Quasicrystals are discrete structures that have highly structured long-range order (represented by pure point or near pure point diffraction) but don't have periodic order. A standard approach to modeling such structures is to take a finite part of it, impose periodic boundary conditions and then apply usual crystallography. Although this type of periodization is used routinely and successfully for many modelling problems in the theory of quasicrystals, it is not entirely satisfactory. Almost periodic order goes beyond periodic order in fundamental ways, its essence appearing as a underlying incommensurability which pervades every part of the theory.

It is possible to construct quasicrystals by means of tiling, fractals and cut-and-project method. The general idea of cut-and-project method is shown in the scheme

$$\begin{array}{ccccc} \mathbb{R}^d & \stackrel{\parallel}{\longleftarrow} & \mathbb{R}^d \times \mathbb{R}^d & \stackrel{\perp}{\longrightarrow} & \mathbb{R}^d \\ & & \cup & & \\ L & \stackrel{1-1}{\longleftarrow} & \widetilde{L} & \stackrel{\text{dense image}}{\longrightarrow} & L' \end{array}$$

Here $\widetilde{L}$ is a lattice in $\mathbb{R}^d \times \mathbb{R}^d$ which is oriented so that the projections into $\mathbb{R}^d$ are $1-1$ and dense.

The left-hand $\mathbb{R}^d$ is *physical space* (where quasicrystal Λ lies).

The right-hand $\mathbb{R}^d$ is *internal space* (to control the projection).

$\tilde{x} \in \widetilde{L}$, $\tilde{x} = (x, x')$ where $x \in L$ and $x' \in L'$.

$(\,\cdot\,)' \; : \; L \to L'$ is defined by $x \mapsto x'$, which passes from physical to internal space.

Window Ω is chosen in internal space (compact, equal to the closure of its interior, and have boundary of measure 0).

Quasicrystal Λ can be defined in the following way $\Lambda(\Omega) := \{x \mid \tilde{x} \in \widetilde{L},\, x' \in \Omega\}$.

We applied the transformation $(\,\cdot\,)'$ to bitmaps and using its discontinuity property we can distribute the lost information evenly throughout the image.

Considering one-dimensional quasi-crystals, we established that some of them can serve as binary keys for the Vernam cipher, while the keys can have an arbitrary length and be uniquely constructed from a small number of integers, namely from the seed point, window length and integer coefficients of a quadratic equation.

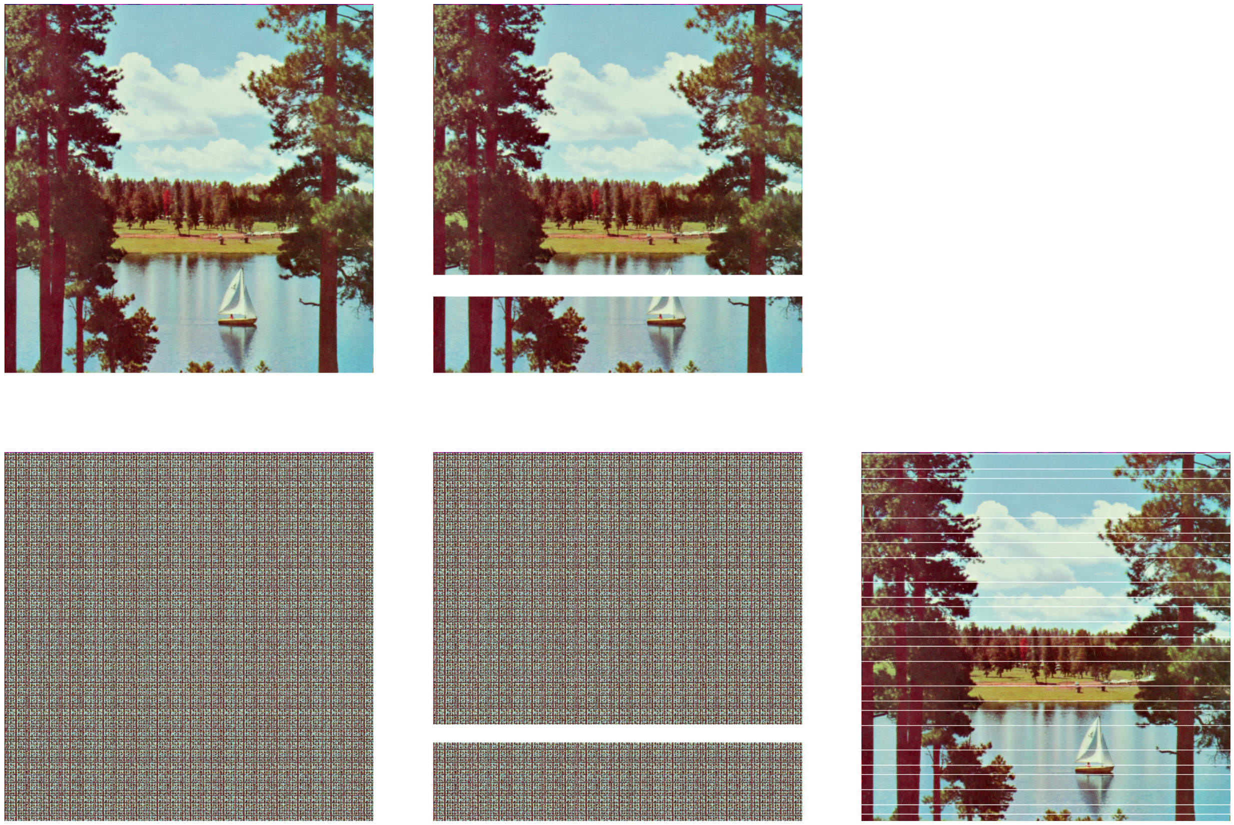

Figure 0.1. An example of information loss during data transmission in the original raster image and in the image encoded with the help of a quasi-crystal.

References

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[2] Nesterenko M., Patera J., Quasicrystal models in cryptography, *AIP Conf. Proc.*, 1191, 148–159, 2009.