Hypercyclicity of symmetric composition operator

Zoriana Novosad
(Lviv University of Trade and Economics, 10, Tuhan-Baranovsky Str., Lviv, Ukraine)
E-mail: zoriana.maths@gmail.com

Andriy Zagorodnyuk
(Vasyl Stefanyk Precarpathian National University, 57 Shevchenka Str., Ivano-Frankivsk, Ukraine)
E-mail: azagorodn@gmail.com

The classical Birkhoff theorem (1929) [1] asserts that any operator of composition with translation

$$x \mapsto x + a,$$

$$T_a \colon f(x) \mapsto f(x + a)$$

is hypercyclic on the space of entire functions $H(\mathbb{C})$ on the complex plane $\mathbb{C}$ if $a \neq 0$. A generalization of the Birkhoff theorem was proved by Godefroy and Shapiro in [2].

Definition 1. Let X be a topological space. A continuous linear operator $T : X \to X$ is said to be *hypercyclic* if there is some vector $x \in X$ such that the set

$$\mathrm{Orb}(T, x) = \{x, Tx, T^2x, \ldots\}$$

of iterates of x is dense in X. The vector x is called a hypercyclic vector associated to the hypercyclic operator T.

The hypercyclicity of a special operator on an algebra of symmetric analytic functions on ℓ_1 was proved in [3]. We construct new class of hypercyclic composition operators on an algebra of symmetric analytic functions on ℓ_1.

References

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[3] Z. Novosad, A. Zagorodnyuk, *Polynomial automorphisms and hypercyclic operators on spaces of analytic functions*, Archiv der Mathematik **89**(2) : 157–166, 2007.