Application of the dynamical system theory for counting black hole entropy of microstates

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Superstring theory is one of the most advanced theories in physics that attempts to unify all four fundamental forces of nature into one single theory. It is based on the idea that all elementary particles and forces in nature can be explained as vibrations of ultramicroscopic strings. Mathematical models in superstring theory have their own unique properties and applications. They make it possible to describe various physical phenomena and processes, such as gravity, electromagnetism, strong and weak nuclear interactions. One of the main properties of mathematical models in superstring theory is their geometric nature. They describe spacetime as a multidimensional space in which strings can move. This allows us to explain many properties of space-time, such as its curvature and topology. The use of mathematical models in superstring theory also makes it possible to study various physical phenomena and processes. For example, they can be used to describe the processes of birth and decay of elementary particles, as well as to explain the properties of black holes and other exotic objects. For example, the Schwarzschild model is used to describe the gravitational field of a black hole

$$ds^2 = -\left(1 - \frac{2MG}{r}\right)dt^2 + \frac{1}{\left(1 - \frac{2MG}{r}\right)}dr^2 + r^2 d\Omega_2^2.$$

This model allows us to describe the properties of a black hole, such as its radius, r, mass, M. Quantum gravity models are also used to explain the properties of black holes. For example, the loop quantum gravity model allows us to describe the properties of black holes at the microscopic level.

Combining quantum mechanics and thermodynamics leads to many hidden degrees of freedom that give a black hole its entropy. These degrees of freedom do not appear in the classical description of black holes and are associated with string theory. The entropy of a black hole from string theory was calculated by Susskind [1]. The calculations of the string entropy is realized through the consideration of a multidimensional lattice of points with the strings inside it, which can move in any of 2d directions. So, the string entropy is

$$S = ln(2d)^n = n \cdot ln2d.$$

Let us consider the use of mathematical models in the aspect of the theory of dynamical systems through the concept of topological entropy to describe chaotic behavior in dynamics, [2]. One can calculate the volume entropy of such space, B,

$$h_v \sim log(VolB)) \sim log(2d)^n.$$

The volume entropy h_v is always bounded above by the topological entropy h_{top} of the geodesic flow on M. Moreover, if M has non-positive sectional curvature, then

$$h_v = h_{top}.$$

From the other hand we know, that

$$h_{top} \geq log|deg(f)|.$$

As the fixed point theorem was proved in 1912 by Brouwer [3], so any continuous mapping of a sphere onto itself has isolated point. Mapping of spaces ($R^{2d} \to R^{2d}$) for ($d = 1, \ldots, n$) can be presented by

$f : S^{2d} \to S^{2d}$ and determines the degree of mapping $degf = 2d$. Therefore, topological entropy of the system of n links of string length, $L = l_s n$, is the n sum,

$$h_{top} = n \cdot log(2d).$$

The Lefschetz number of one link

$$L(f^n) = 1 + (-1)^m degf^n$$

is equal to

$$L(f^n) = 1 + (2d)^n.$$

So, according to the Lefschetz formula we can calculate the index of isolated point on manifold M

$$L(f) = \sum ind_f x.$$

Considering the Hopf-Poincare theorem

$$\sum ind_f x = \chi(M)$$

we can receive the following formula

$$1 + ((2d)^n)^n = \chi(M).$$

Thus, using the theory of dynamical systems, we calculated the entropy, Lefschetz number and Euler characteristic of a black hole, represented as a multidimensional cubic space.

References

[1] Susskind Lectures on String Theory. July 10, 2022 version 2.1. URL: https: www.umsl.edu/∼ *majzoube/docs/string_theory_basics_v2.2.pdf*.

[2] Katok A., Hasselblatt B. Introduction to the modern theory of dynamical systems. Cambridge University Press 1995, 824 p.

[3] Brouwer L.E.J. Uber Abbildung von Mannigfaltigkeiten. *Mathematische Annalen*, Volume: 71, page 97-115, 1912.