

# EINSTEIN SOLVMANIFOLDS NOT BASED ON NILSOLITONS

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In this seminar, we describe different techniques to construct pseudo-Riemannian Einstein solvmanifolds, expanding beyond the traditional framework reliant on nilsolitons.

In the first part, we review Einstein solvmanifolds and their construction based on nilsolitons. We will recall the notion of pseudo-Iwasawa and the role of nice nilpotent Lie algebras. Subsequently, we present two different constructions of Einstein solvmanifolds that do not rely on nilsolitons and are peculiar to the indefinite case. The first construction uses contact symplectic reduction (a peculiar feature of pseudo-Sasaki geometry). The second, which is quite new, is based on solving the generalized nilsoliton equation and introduces a new methodology. Both constructions yield examples that are not isometric to any Einstein solvmanifold of pseudo-Iwasawa type.

We will also discuss related geometric structures, such as Sasaki, pseudo-Kähler, and para-Kähler geometries. Time permitting, we will explore open pathways for further research in differential geometry.

This talk is based on joint work with D. Conti and R. Segnan Dalmasso.

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