ON THE MAPPING OF SURFACES OF EUCLIDEAN SPACES

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Let us consider the Euclidean spaces E_4 and $\overline{E}_4$ as completely orthogonal subspaces in the proper Euclidean space E_8, having one common point O. Let V_2 and $\overline{V}_2$ be smooth surfaces in E_4 and $\overline{E}_4$ respectively.

We will study differentiable one-to-one mapping T of a domain $\Omega \subset V_2$ onto a domain $\overline{\Omega} \subset \overline{V}_2$. If a point X_1 inscribes a domain Ω, and $X_2 = T(X_1) \subset \overline{\Omega}$, then a point X with radius vector $\overrightarrow{X} = \overrightarrow{X}_1 + \overrightarrow{X}_2$ inscribes a certain two-dimensional surface V_2^*, called the graph of the mapping T [1].

In [2], [3], [4], it is shown that in this case, each surface V_2 and $\overline{V}_2$, there arise orthogonal sets $\delta_2 \subset V_2$ and $\overline{\delta}_2 \subset \overline{V}_2$.

The following theorems proved

Theorem 1. *The sets δ_2 and $\overline{\delta}_2$ correspond to the mapping T if and only if one of the following conditions is satisfied:*

1) the sets δ_2 and $\overline{\delta}_2$ coincide with the base of the mapping T,

2) the mapping T is conformal.

Theorem 2. *If the surfaces V_2 and $\overline{V}_2$ carry conjugate sets and these sets correspond, then the sets δ_2 and $\overline{\delta}_2$ serve as the basis of the mapping T if and only if the condition.*

$$\overrightarrow{C}_{12}\left[\left(C_{12}^4\overline{\gamma}^{1i} - C_{12}^3\overline{\gamma}^{2i}\right)\overrightarrow{e}_{4+i}\right] = 0$$

is satisfied.

Theorem 3. *The base of the mapping T harmonically separates the conjugate sets Σ_2 and $\overline{\Sigma}_2$ if and only if condition $\overrightarrow{C}_{12}\left(C_{12}^3\overrightarrow{e}_1 - C_{12}^4\overrightarrow{e}_2\right) = 0$ is satisfied.*

Theorem 4. *A pair of surfaces V_2, $\overline{V}_2$, carrying conjugate sets corresponding to the mapping T is determined by specifying four functions of two arguments.*
Note that an arbitrary pair of surfaces V_2, $\bar{V}_2$, is defined by specifying six functions of two arguments (two functions for each of the surfaces $V_2 \subset E_4$ and $\bar{V}_2 \subset E_4$- and two functions for specifying the mapping $T: \Omega \to \bar{\Omega}$).

Theorem 5. *If the surfaces V_2 and $\overline{V}_2$ carry orthogonal conjugate networks and these networks correspond, then the networks δ_2 and $\overline{\delta}_2$ correspond in this mapping T if and only if one of the following conditions is satisfied:*

1) $C_{12}^3 = 0, C_{12}^4 \neq 0$ (or $C_{12}^4 = 0, C_{12}^3 \neq 0$). Here C_{12}^3, C_{12}^4 do not vanish simultaneously, since rang $\left\|C_{ij}^n\right\| = 3$. Geometrically, this means that the vector $\overrightarrow{C}_{12}$ is either collinear with the vector $\overrightarrow{\mathcal{E}}_3$, or with $\overrightarrow{\mathcal{E}}_4$.

2) The mapping T is conformal. Considering that the vector $\overrightarrow{C}_{12}$ is the following decomposition.

$$\overrightarrow{C}_{12} = \overrightarrow{m} - \overrightarrow{\overline{m}}$$

we have

Corollary 6. *Let* $\overline{\Sigma}_2 = T\left(\Sigma_2\right)$ *and let the sets* Σ_2 *and* $\overline{\Sigma}_2$ *be orthogonal and conjugate. The sets* Σ_2^* *of the graph* V_2^* *is a set of curvature lines with respect to the mean normal if and only if*

$$\overrightarrow{\mu}^* \cdot \overrightarrow{m} = \overrightarrow{\mu}^* \cdot \overrightarrow{\overline{m}}$$

where $\overrightarrow{\mu}^*$ *is the mean normal vector of the surface* V_2^*.

References

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