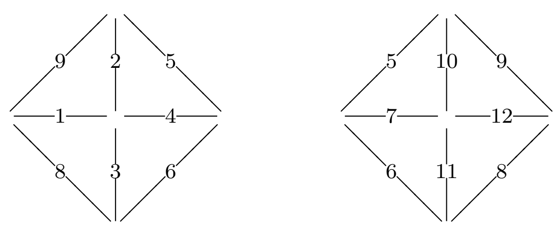
Deformations, fundamental groups, and Zariski pairs in classification of algebraic curves and surfaces

Meirav Amram
(SCE, Israel)
E-mail: meiravt@sce.ac.il

Classification of algebraic surfaces and curves has been a major mathematical problem over the years. In the talk I will focus on both studies. The background and some new results will be presented, especially by using topological and algebraic methods in geometry.

Classification of algebraic surfaces: Studying how algebraic surfaces change into unions of planes is a fascinating area of research. These changes, or deformations, help to understand the geometry and topology of these surfaces, especially by looking at singular points and invariants like fundamental groups. Planar and non-planar deformations of algebraic surfaces involve breaking down these surfaces into unions of planes. Non-planar deformations are more complex than the planar ones because they involve the connection of edges to form high multiple singularities. Both types are crucial for understanding the geometry and topology of these surfaces, with applications in algebraic geometry, topology, and physics.

In the following figure we can see one example of a non-planar deformation, one of many that are of great interest to mathematicians in topology and algebraic geometry. If we glue the two pieces in the figure along their external edges, we will get a non-planar deformation, with complicated singularities along this gluing.



We can compute the fundamental group G of the complement of the branch curve of an algebraic surface. The fundamental group is an invariant of the surface. Via the deformation, we can derive the dual graph that represents the group and contributes a lot of information to the classification.

If group G is complicated, we can compute the fundamental group of the Galois cover of the surface, and it is an invariant of the surface as well.

We give new results in this area of research; selected references are [1]-[6].

Classification of algebraic curves: We classify algebraic curves using Zariski pairs, which are pairs of curves that have the same combinatorial structure but differ in their topological properties. By studying these pairs, we gain insight into the unique characteristics of each curve.

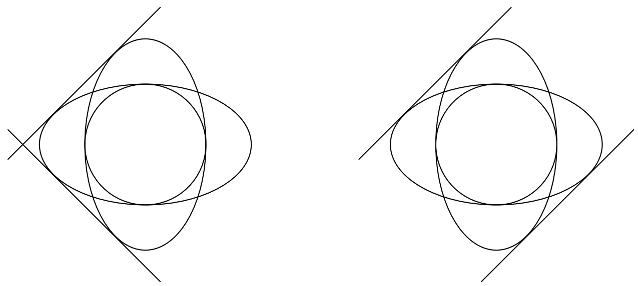
Our research focuses on line arrangements and conic-line arrangements. The deformations of these arrangements are interesting objects by themselves, and the study helps us to see how different changes in the curves affect the fundamental groups related to them. The computations give us a better understanding of their underlying topology.

Concerning line arrangements: Zariski pairs of line arrangements cannot be in the same component of the moduli space. This is a powerful tool to study the moduli space of line arrangements.

Concerning conic-line arrangements: there are no Zariski pairs of degree ≤ 5. For degree 7, there are already some interesting examples of Zariski pairs.

We will see the correspondence between curves and fundamental groups and understand the rules given by Zariski pairs.

In the following figure we give an example of a Zariski pair of degree 8. Such an example will be explained later, among other examples from [7]-[10].



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