# On topologization of subsemigroups of the bicyclic monoid

**Adriana Chornenka**
(Ivan Franko National University of Lviv, Universytetska 1, Lviv, 79000, Ukraine)
*E-mail:* adriana.chornenka@lnu.edu.ua

**Oleg Gutik**
(Ivan Franko National University of Lviv, Universytetska 1, Lviv, 79000, Ukraine)
*E-mail:* oleg.gutik@lnu.edu.ua

In this paper we shall follow the terminology of [2, 5, 6, 7].

A semigroup $S$ is called *inverse* if for any element $x \in S$ there exists a unique $x^{-1} \in S$ such that $xx^{-1}x = x$ and $x^{-1}xx^{-1} = x^{-1}$. The element $x^{-1}$ is called the *inverse of* $x \in S$.

A topology $\tau$ on a semigroup $S$ is called a *semigroup* (*shift-continuous*) topology if $(S, \tau)$ is a topological (semitopological) semigroup.

The bicyclic monoid $\mathcal{C}(p, q)$ is the semigroup with the identity 1 generated by two elements $p$ and $q$ subjected only to the condition $pq = 1$. The bicyclic monoid admits only the discrete semigroup Hausdorff topology [4]. Bertman and West in [1] extended this result for the case of Hausdorff semitopological semigroups. $T_1$-topologizations of the bicyclic monoid $\mathcal{C}(p, q)$ are studied in [3].

**Theorem 1.** *Let $S$ be a subsemigroup of the bicyclic semigroup $\mathscr{C}(p, q)$. If $S$ contains infinitely many idempotents then every shift-continuous Hausdorff topology on $S$ is discrete.*

**Corollary 2.** *Let $S$ be an inverse subsemigroup of the bicyclic semigroup $\mathscr{C}(p, q)$. Then every shift-continuous Hausdorff topology on $S$ is discrete.*

Also we give sufficient algebraic conditions on a subsemigroup $S$ of the bicyclic semigroup $\mathscr{C}(p, q)$ when the semigroup $S$ admits a non-discrete Hausdorff semigroup (shift-continuous) topology.

## References

[1] M. O. Bertman and T. T. West, *Conditionally compact bicyclic semitopological semigroups*, Proc. Roy. Irish Acad. **A76** (1976), no. 21–23, 219–226.

[2] J. H. Carruth, J. A. Hildebrant, and R. J. Koch, *The theory of topological semigroups*, Vol. I, Marcel Dekker, Inc., New York and Basel, 1983.

[3] A. Chornenka and O. Gutik, *On topologization of the bicyclic monoid*, Visn. L'viv. Univ., Ser. Mekh.-Mat. **95** (2023), 46–56.

[4] C. Eberhart and J. Selden, *On the closure of the bicyclic semigroup*, Trans. Amer. Math. Soc. **144** (1969), 115–126.

[5] R. Engelking, *General topology*, 2nd ed., Heldermann, Berlin, 1989.

[6] M. Lawson, *Inverse semigroups. The theory of partial symmetries*, Singapore: World Scientific, 1998.

[7] W. Ruppert, *Compact semitopological semigroups: an intrinsic theory*, Lect. Notes Math., **1079**, Springer, Berlin, 1984.