Isoperimetric profile and quantitative orbit equivalence for lamplighter-like groups (joint work with Vincent Dumoncel)

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Measure equivalence has been introduced by Gromov as a measured analogue of quasi-isometry. In this talk we focus on the closely related notion of orbit equivalence which is in fact a source of examples for measure equivalence.

Two groups G and H are orbit equivalent if there exist two free probability measure-preserving G- and H-actions on a standard probability space, having the same orbits.

However Orstein and Weiss proved that two infinite amenable groups are orbit equivalent. To get an interesting theory, we need to strengthen the definition of orbit equivalence.

In this talk, we introduce the quantitative versions of orbit equivalence, which propose to add some restrictions on two maps called *cocycles*, which describe more precisely the orbit equalities of a given orbit equivalence between G and H. Given maps $\varphi, \psi \colon \mathbb{R}_+ \to \mathbb{R}_+$, we define the notion of φ-integrability of a cocycle (for instance, being L^p when $\varphi(x) = x^p$), and the notion of (φ, ψ)-integrability for an orbit equivalence (which asks that one cocycle is φ-integrable and the other is ψ-integrable). If G and H are amenable, these quantifications provide interesting information on their geometry, since the *isoperimetric profiles* of the groups give obstructions to the existence of quantitative versions of orbit equivalence (see [1, Theorems 1.1, Corollary 4.7]), and then lead to the following problem.

Problem 1. What is the "highest" map $\varphi \colon \mathbb{R}_+ \to \mathbb{R}_+$ such that there exists a (φ, L^0)-integrable orbit equivalence from G to H, and vice versa?

In some sense, this is a more quantitative comparison between groups. The highest quantification we can get answers to the following question: if two groups are not quasi-isometric, how much do their geometry differ?

In a joint work with Vincent Dumoncel, we study quantitative orbit equivalence for *lampshuffler groups*. Given a group H, the lampshuffler group over H is

$$\mathsf{Shuffler}(H) := \mathrm{FSym}(H) \rtimes H,$$

where $\mathrm{FSym}(H)$ is the set of permutations of H of finite support, and the action of H on it is given by $k \cdot \sigma \colon h \in H \to k\sigma(k^{-1}h)$.

Lampshufflers belong to a large class of groups which look like *lamplighter group*. They have been intensively studied in [2], where the authors found conditions for two lamplighters to be quasi-isometric, for two lampshufflers to be quasi-isometric, etc.

Outlines of our work: Our goal is to quantitatively compare lampshufflers. We first build explicit orbit equivalence couplings between lampshufflers and quantify the associated cocycles. Secondly, we compute the isoperimetric profiles of lampshufflers to prove that the quantifications we find are optimal. In this talk, I will present our results with more details.

References

[1] T. Delabie, J. Koivisto, F. Le Maître, and R. Tessera. Quantitative measure equivalence between amenable groups. *Annales Henri Lebesgue*, 5:1417–1487, 2022.
[2] A. Genevois, R. Tessera. Lamplighter-like geometry of groups. ArXiv, 2024.