LOCAL MODULI OF SASAKI-EINSTEIN METRICS ON RATIONAL HOMOLOGY 7-SPHERES

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For Sasakian manifolds, which are roughly speaking the odd dimensional analogue of Kähler manifolds, the moduli problem has been addressed and an appropriate notion of moduli space has been achieved, see [1] and references therein. In particular, finding the number of components of this moduli has been studied and used to obtain lower bounds for the dimension of the moduli space for links of Brieskorn-Pham polynomials and Smale manifolds [4, 6, 2, 7]. An important ingredient to obtain information on the moduli is given by the use of invertible polynomials, particularly to describe the local moduli of Sasaki-Einstein metrics.

In this talk, which is based on a joint work with J. Lope [10], we determine the dimension of the local moduli space of Sasaki-Einstein metrics for links of invertible polynomials coming from the list of Johnson and Kollár of anticanonically embedded Fano 3-folds of index 1 [5] that produce \mathbb{Q} -homology 7-spheres, that is, 7-manifolds whose \mathbb{Q} -homology equals that of the 7-sphere [3, 8]. In order to do so, we propose additional conditions to the Diophantine equations associated to this problem. We also find solutions for the problem associated to the moduli for the Berglund-Hübsch duals [9] of links arising from Thom-Sebastiani sums of chain and cycle polynomials.

Our findings can be interpreted in two different settings:

- Seifert S¹-bundles are Q-homology spheres if and only if the corresponding orbifolds are Q-homology complex projective spaces, so our results describes some components of the moduli space of Q-homology complex projective 3-spaces with quotient singularities.
- Sasaki-Einstein structures on the manifold determine Ricci-flat Kähler cone metrics on the corresponding affine cone, so our results give information on the moduli of Calabi-Yau cones.

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