Bonded Knots and Braids

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Proteins, fundamental to biological function, are complex molecules composed of amino acid chains that fold into highly specific three-dimensional configurations. These folded structures are stabilized by intramolecular bonds—interactions between distant residues, that are essential for maintaining shape and functionality. Mathematically, such structures can be modeled as bonded knots, where the protein backbone forms a knot or open curve, and the stabilizing interactions are represented by bonds connecting non-adjacent segments ([2, 3]).

In this talk, I will present the theory of bonded knots and its extension to bonded braids, emphasizing their structural, topological, and algebraic features. Bonded knots ([1]) generalize classical knot theory by introducing bond constraints, which fall into three main classes: long bonds (topological and rigid-vertex), regular bonds (with unknotted connections), and tight bonds (modeled as non-crossing line segments). For each type, I will describe a system of Reidemeister-type moves—both in the topological and rigid frameworks—and introduce core invariants that classify these objects.

I will then transition to bonded braids ([4]), discussing an Alexander-type theorem that relates bonded knots to their braid representations in the topological category. The talk will include the definition of the bonded braid monoid, its generating set and relations, and a Markov-type theorem capturing braid equivalence. I will also sketch how this monoid embeds into a group, revealing deeper algebraic structure.

Time permitting, I will conclude with a look at bonded knots on the torus and their relation to doubly periodic bonded tangles ([5, 6, 7]), offering insights into their covering spaces and potential relevance to structural biology. This presentation provides an accessible entry point into the emerging theory of bonded knots and braids and its rich connections to topology, algebra, and the geometry of biomolecular systems.

This is a joint work with Prof. Dr. L.H. Kauffman (University of Illinois at Chicago, U.S.A.), Prof. Dr. Sofia Lambropoulou (National Technical University of Athens, Greece) and Dr. Sonia Mahmoudi (Tohoku University, Japan).

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