Spray-Invariant Sets in Infinite-Dimensional Manifolds

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This talks investigates the behavior of geodesics within subsets of infinite-dimensional manifolds, including singular spaces such as stratified spaces. We define *spray-invariant* sets as those where any geodesic starting within the set remains entirely within it. The regularity of these sets significantly impacts their geometric properties.

We study these sets in the context of spray geometry, without relying on Finsler or Riemannian metrics, enabling the analysis of geodesic dynamics in infinite-dimensional manifolds where traditional geometric tools may not be applicable. For a subset S of a manifold M and a spray $\mathbf{S}$ on M, we define an *admissible set* $A_{\mathbf{S},S}$ that characterizes when a geodesic remains within S. We prove that if S is closed, then a geodesic lies entirely in S if and only if its tangent vector belongs to $A_{\mathbf{S},S}$ for all time, establishing $A_{\mathbf{S},S}$ as a key invariant. For sufficiently differentiable submanifolds S, we show that $A_{\mathbf{S},S}$ characterizes totally geodesic submanifolds.

We also show that spray-invariant sets remain invariant under spray automorphisms. We explore the relationship between spray invariance and the tangency of the spray to the admissible set, addressing this using the Nagumo-Brezis Theorem, where we establish the equivalence between spray invariance and this tangency condition.

Finally, we study Lie group actions on Banach manifolds and their orbit type decompositions. We prove that if the action admits suitable local slices (defined by invariance, local triviality, and transversality), then each orbit type stratum is invariant under a group-invariant spray.

References

[1] Kaveh EFtekharinasab. Spray-Invariant Sets in Infinite-Dimensional Manifolds1. *Arxiv:2505.10980*, 2025.