Szegő and Poisson kernels on Grauert Tubes and Lie group actions

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Let (M, κ) be a compact, connected, real-analytic Riemannian manifold. It is well known that M can be complexified in an essentially unique way, and that on a tubular neighborhood of M inside the complexification, there exists a Kähler structure compatible with the metric κ, with Kähler potential given by a real-analytic, strictly plurisubharmonic, and positive exhaustion function ρ. The sublevel sets $\rho < \tau^2$ are known as open Grauert tubes of radius τ, and they are strictly pseudoconvex domains in the complexification. Their boundaries, denoted X^τ, inherit a CR-holomorphic structure of codimension 1 and a natural contact structure.

If (M, κ) is endowed with an isometric action of a compact, connected Lie group G, this action lifts to a holomorphic action on the open Grauert tube and to a CR action on the boundary X^τ, both commuting with the Hamiltonian flow of ρ, known as the geodesic flow. These lifted actions give rise to unitary representations, respectively, on the eigenspaces of the Laplacian on (M, κ) and on the eigenspaces of an elliptic self-adjoint Toeplitz operator induced by the generator of the homogeneous geodesic flow on the the Hardy space $H(X^\tau)$.

This talk, based on joint work with R. Paoletti, has a twofold aim: first, to describe the scaling asymptotics of the equivariant Poisson-wave kernel, which relates to the asymptotic concentration of complexified eigenfunctions of the Laplacian in a fixed isotype, when restricted to X^τ; and second, to describe the scaling asymptotics of the equivariant Szegő kernel, which pertains to the asymptotic concentration of the eigenfunctions of the aforementioned Toeplitz operator in a given isotypical component.