WEIGHTED INEQUALITIES FOR SUB-MONOTONE FUNCTIONALS AND APPLICATIONS

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In this talk, we will briefly discuss the history of weighted Hardy inequalities, and present new elementary and universal proof of the well known two weighted Hardy inequality from resent paper [1]. In the second part we present (see [2]) a set of relations between several quite diverse types of weighted inequalities involving various integral operators and fairly general quasi-norm-like functionals, which we call sub-monotone. The main result enables one to solve a specific problem by transferring it to another one for which a solution is known. Inequalities for Hardy, Copson, geometric mean and harmonic mean operators are shown to be interlinked. We give applications weighted inequalities restricted to cones of monotone functions [3].

References

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