On Collectively AM, b-AM, KB, quasi-KB sets of operators

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A linearly ordered real vector space E is called a vector lattice if $sup\{x, y\}$ exists in E for every $x, y \in E$. Let E be a vector lattice. For each $x, y \in E$ with $x \leq y$, the set $[x, y] = \{z \in E : x \leq z \leq y\}$ is said to be an order interval. A subset A of E is called order bounded if it is included in some order interval. By E^+ we denote the set of all positive elements in E. A Banach space $(E, \|.\|)$ is called a Banach lattice if E is a vector lattice and its norm satisfies the following property: for each $x, y \in E$ such that $|x| \leq |y|$, we have $\|x\| \leq \|y\|$.

Definition 1. A net (x_α) in an Archimedean vector lattice E is called order convergent to $x \in E$ if there exists a net (y_β) satisfying $y_\beta \downarrow 0$, and for any β there exists α_β such that $|x_\alpha - x| \leq y_\beta$ for all $\alpha \geq \alpha_\beta$.

Definition 2. An operator T from a vector lattice E into a Banach lattice F is called AM-compact if the image of each order bounded subset of E is relatively compact in F.

Every regular compact operator is an AM-compact. The identity operator $I : l_1 \to l_1$ is an AM-compact operator, but it is not compact operator.

Definition 3. Let $\tau \subset L(X, Y)$. τ is called collectively AM-compact if for every order interval $[x, y]$ in X the set $\tau[x, y] = \bigcup_{T \in \tau} T[x, y]$ is relatively compact.

Definition 4. Let A and B be subsets of $L^+(X, Y)$. A is called dominated by B if, for each $S \in A$ there exists $T \in B$ such that $S \leq T$.

Definition 5. Let E, F be normed lattices and $T : E \to F$ is called KB-operator for every bounded increasing sequence (x_n) in E^+, there is an $x \in E$ such that (Tx_n) converges to Tx in norm.

Definition 6. Let E, F be normed lattices and $\tau \subseteq L(E, F)$. We say that τ is a collectively KB set of operators if, for every increasing bounded sequence (x_n) in E^+, there is an indexed subset $\{x_T\}_{T \in \tau}$ of E satisfying $\{(Tx_n) : T \in \tau\}$ norm converges to $\{Tx_T\}_{T \in \tau}$.

A collectively AM-compact($b - AM$ compact, KB)- set of operators is a generalization of the AM-compact($b - AM$ compact,KB) and quasi KB-operator. We investigate collective versions of some operators such as AM-compact, $b - AM$ compact, KB and quasi-KB operators.

We discuss the domination problem for collectively AM-compact,$b - AM$ compact, KB and quasi-KB sets of operators.

For this subject, we give the following references.

References

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