# The approximate solution of the Boltzmann equation for the hard sphere model

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The Boltzmann kinetic equation plays an important role in the kinetic theory of gases. It is describes the evolution of rarefied gases. For the hard sphere model, the equation has the form [1]

$$D(f) = Q(f,f); \tag{1}$$

$$D(f) \equiv \frac{\partial f}{\partial t} + \left(V, \frac{\partial f}{\partial x}\right), \tag{2}$$

$$Q(f,f) \equiv \frac{d^2}{2} \int_{R^3} dV_1 \int_{\Sigma} d\alpha |(V - V_1, \alpha)| \Big[ f(t,x,V_1') f(t,x,V') - f(t,x,V) f(t,x,V_1) \Big]. \tag{3}$$

The only exact solution to equation (1), which is known explicitly up to now, is the Maxwell distribution $M$ or simply Maxwellian (after J. C. Maxwell, Scottish physicist). It makes both parts of the Boltzmann equation equal to zero, namely

$$D(M) = 0, \quad Q(M,M) = 0. \tag{4}$$

The solution to this equation (1)-(3) will be look for in the next form[2]

$$f(t,x,V) = \sum_{i=1}^{\infty} \varphi_i(t,x) M_i(t,x,V), \tag{5}$$

where coefficient functions $\varphi_i(t,x)$ are nonnegative smooth functions on $\mathbb{R}^4$. $M_i$ are Maxwellians (4), which describe the eddy-like motion of the gas.

As a measure of the deviation between the parts of equation (1) we will consider a uniform-integral error of the form:

$$\Delta = \sup_{(t,x)\in\mathbb{R}^4} \int_{\mathbb{R}^3} \big| D(f) - Q(f,f) \big| dV. \tag{6}$$

In the paper [2], we were obtained sufficient conditions for the coefficient functions and hydrodynamic parameters appearing in the distribution, which enable one to make the analyzed error (6) as small as desired.

## References

[1] S. Chapman and T.G. Cowling. The Mathematical Theory of Non-Uniform Gases. *Cambridge Univ. Press,* Cambridge, 1952.

[2] O. O. Hukalov and V. D. Gordevskyy. *The Interaction of an Infinite Number of Eddy Flows for the Hard Spheres Model, J. Math. Phys. Anal. Geom.* Vol. 17, No. 2, 2021, 163–174.