Analysis of Weak Associativity in Some Hyper-Algebraic Structures that Represent Dismutation Reactions

Kehinde Gabriel Ilori
(Department of Mathematics and Statistics, Redeemer's University Ede, Nigeria.)
E-mail: kennygilori@gmail.com

Temitope Gbolahan Jaiyeola
(Department of Mathematics, University of Lagos, Akoka, Nigeria.)
E-mail: tgjaiyeola@unilag.edu.ng

Oyeyemi Oluwaseyi Oyebola
(Department of Mathematics and Computer Science, Brandon University, Brandon Manitoba, Canada.)
E-mail: oyebolao@brandonu.ca

In this paper, some chemical systems of Tin (Sn), Indium (In) and Vanadium (V) which are represented by hyper-algebraic structures $(S_{Sn}, \oplus)$, $(S_{In}, \oplus)$ and $(S_V, \oplus)$ were studied. The analyses of their algebraic properties and the probabilities of elements in dismutation reactions were carried out with the aid of computer codes in Python programming language. It was shown that in the dismutation reactions, the left nuclear (N_λ)-probability, middle nuclear (N_μ)-probability and right nuclear (N_ρ)-probability for each of the hyper-algebraic structures $(S_{Sn}, \oplus)$, $(S_{In}, \oplus)$ and $(S_V, \oplus)$ is less than 1.000. This implies that, $(S_{Sn}, \oplus)$, $(S_{In}, \oplus)$ and $(S_V, \oplus)$ are non-associative hyper-algebraic structures. Also, from the results obtained for FLEX-probability, it was shown that, $(S_{Sn}, \oplus)$, $(S_{In}, \oplus)$ and $(S_V, \oplus)$ have flexible elements because the values of their FLEX-probabilities are 1.000 each. Hence, $(S_{Sn}, \oplus)$, $(S_{In}, \oplus)$ and $(S_V, \oplus)$ are flexible. Overall, $(S_V, \oplus)$ exhibited the lowest measure of weak-associativity, $(S_{Sn}, \oplus)$ exhibited lower measure of weak-associativity, and $(S_{In}, \oplus)$ exhibited a low measure of weak-associativity.

Definition 1. (Semihypergroup, Quasihypergroup, Hypergroup, H_v-group)

An hypergroupoid or polygroupoid $(H, \circ)$ is the pair of a non-empty set H with an hyperoperation $\circ : H \times H \to P(H)\backslash\{\emptyset\}$ defined on it.

An hypergroupoid $(H, \circ)$ is called a semihypergroup if

(i): it obeys the associativity law $a \circ (b \circ c) = (a \circ b) \circ c$ for all $a, b, c \in H$, which means that

$$\bigcup_{u \in\ a \circ b} u \circ c = \bigcup_{v \in\ b \circ c} a \circ v$$

An hypergroupoid $(H, \circ)$ is called a quasihypergroup if

(ii): it obeys the reproduction axiom $x \circ H = H = H \circ x$ for all $x \in H$.

An hypergroupoid $(H, \circ)$ is called an hypergroup if it is a semihypergroup and a quasihypergroup.

A hypergroupoid $(H, \circ)$ is called an H_v-semigroup it obeys the weak associativity (WASS) condition

(iii): $x \circ (y \circ z) \cap (x \circ y) \circ z \neq \emptyset$ for all $x, y, z \in H$.

A hypergroupoid $(H, \circ)$ is called an H_v-group if it is a quasihypergroup and a H_v-semigroup.

According to Davvaz et al. [1], all combinational probabilities for the set $S_{Sn} = \{Sn, Sn^{2+}, Sn^{4+}\}$ without energy can be displayed as follows in Table 1.1 and the major products are shown. $(S_{Sn}, \oplus)$ is not a quasihypergroup, not a semihypergroup, not a hypergroup and not an H_v-group. But it is an H_v-semigroup. Summarily, even though $(S_{Sn}, \oplus)$ is not associative and has weak associativity, it is commutative. However, it has an hyper-substructure that is associative, i.e. an hypergroup.

Table 1.1. Dismutation reaction for tin (Sn)

$\oplus$	Sn	Sn^{2+}	Sn^{4+}
Sn	Sn	Sn, Sn^{2+}	Sn^{2+}
Sn^{2+}	Sn, Sn^{2+}	Sn^{2+}	Sn^{2+}, Sn^{4+}
Sn^{4+}	Sn^{2+}	Sn^{2+}, Sn^{4+}	Sn^{4+}

Definition 2. (Left nuclear element)

Let $(P, \cdot)$ be a polygroupoid. The left nucleus pair of $x \in P$ will be denoted by $N_\lambda(x)$ and defined as $N_\lambda(x) = \{(y, z) \in P \times P \mid x \cdot (yz) = (xy) \cdot z\}$. $x \in P$ will be said to be left nuclear if $N_\lambda(x) = P \times P$.

Definition 3. (Probability of left nuclear element/polygroupoid)

Let $(P, \cdot)$ be a polygroupoid.

(1) The probability of an element $x \in P$ being left nuclear will be denoted by $Pr_{N_\lambda(P,\cdot)}(x)$ and will be defined as $Pr_{N_\lambda(P,\cdot)}(x) = \dfrac{|N_\lambda(x)|}{|P|^2}$.

(2) The probability of $(P, \cdot)$ being left nuclear will be denoted by $Pr_{N_\lambda}(P, \cdot)$ and defined as $Pr_{N_\lambda}(P, \cdot) = \dfrac{\sum_{x\in P} Pr_{N_\lambda(P,\cdot)}(x)}{|P|}$.

Based on our Definition 2 and Definition 3, we have the result below.

Lemma 4. *Let $(P, \cdot)$ be a polygroupoid. Let the left nucleus of $(P, \cdot)$ be defined as $N_\lambda(P, \cdot) = \{x \in P \mid x \cdot (yz) = (xy) \cdot z \ \forall \ (y, z) \in P \times P\}$. Then:*

(1) $N_\lambda(P, \cdot) = \{x \in P \mid N_\lambda(x) = P \times P\} = \{x \in P \mid x \text{ is left nuclear}\}$.

(2) $$Pr_{N_\lambda}(P, \cdot) = \frac{\sum_{x\in P} |N_\lambda(x)|}{|P|^3}.$$

Using Lemma 4, the results in Table 4.2 were gotten using the information in Table 1.1.

Table 4.2. Probability of elements in dismutation reaction tin, S_{Sn}

Probability of Properties	Sn	Sn^{2+}	Sn^{4+}	S_{Sn}
Left Nucleus $P_{N_\lambda}(\cdot)$	0.5556	0.7778	0.5556	0.6297
Middle Nucleus $P_{N_\mu}(\cdot)$	0.5556	0.7778	0.5556	0.6297
Right Nucleus $P_{N_\rho}(\cdot)$	0.5556	0.7778	0.5556	0.6297
Flexibility $P_{FLEX}(\cdot)$	1.0000	1.0000	1.0000	1.0000
Left Alternative Property $P_{LAP}(\cdot)$	0.6667	1.0000	0.6667	0.7778
Right Alternative Property $P_{RAP}(\cdot)$	0.6667	1.0000	0.6667	0.7778

References

[1] B. Davvaz, A. Dehghan-Nezhad and A. Benvidi. Chemical hyperalgebra: dismutation reactions. *MATCH Commun. Math. Comput. Chem*, 67: 55–63, 2012.

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