On special affinor structures on the Gaudi's surface

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We investigated the existence of a special affinor structure on the surface, which is named after the Catalan architect Antonio Gaudí.

The Gaudí's surface is given in R_3 by the general equation $z = kxsin(\frac{y}{a})$, were k, a - some constants, or by the parametric equations

$$x = u^1, \quad y = u^2, \quad z = ku^1 sin(\frac{u^2}{a}).$$

As is known [2] an affinor structure $F_i^h(x)$ in Riemannian space $(V_n(x), g_{ij}(x))$ that satisfies condition

$$F_\alpha^h F_i^\alpha = e\delta_i^h, \qquad e = 0, \pm 1, \tag{1}$$

$$i, h, j, ... = 1, 2, ...n,$$

is called

- *elliptic* if $e = -1$,
- *hyperbolic* if $e = +1$,
- *m-parabolic* when $e = 0$, $rankF = m \ \ (2m < n)$,
- *parabolic* when $e = 0$, $rankF = m \ \ (2m = n)$.

Usually the affinor structure F_i^h is coordinated with the Riemannian metric g_{ij} as follows

$$g_{i\alpha} F_j^\alpha = -g_{j\alpha} F_i^\alpha. \tag{2}$$

So, we are looking for an affinor structure $F_i^h(u)$ on the Gaudi's surface $(V_2^G(u), g_{ij}(u))$, $i, j, h = 1, 2$ provided that $a = k = 1$, that satisfies conditions (1),(2). Then

$$(g_{ij}(u)) = \begin{pmatrix} 1 + Sin^2 u^2 & u^1 Cos u^2 Sin u^2 \\ u^1 Cos u^2 Sin u^2 & 1 + (u^1)^2 Cos^2 u^2 \end{pmatrix},$$

As a result, it turned out that the Gaudi's surface does not admit an affinor e-structure of hyperbolic and parabolic types, but it admits an elliptic affinor structure

$$(F_i^h(u)) = \begin{pmatrix} \frac{-u^1 Cos u^2 Sin u^2}{1+sin^2u^2+(u^1)^2Cos^2u^2} & \frac{-1-(u^1)^2Cos^2u^2}{1+sin^2u^2+(u^1)^2Cos^2u^2} \\ \frac{1+Sin^2u^2}{1+sin^2u^2+(u^1)^2Cos^2u^2} & \frac{u^1 Cos u^2 Sin u^2}{1+sin^2u^2+(u^1)^2Cos^2u^2} \end{pmatrix},$$

which is necessarily absolutely parallel:

$$F_{i,j}^h = 0.$$

Here comma «,» is a sign of the covariant derivative in respect to the connection of V_2^G, that is the Gaudi's surface admits a Kähler structure [1].

In this case, the corresponding fundamental 2-form has the form

$$(F_{ij}(u)) = (g_{i\alpha}(u) F_j^\alpha(u)) = \begin{pmatrix} 0 & -(1 + sin^2u^2 + (u^1)^2 Cos^2 u^2)^{0,5} \\ (1 + sin^2u^2 + (u^1)^2 Cos^2 u^2)^{0,5} & 0 \end{pmatrix}.$$

References

[1] Sinyukov N.S. *Geodesic mappings of Riemannian spaces.*. M.: Nauka, Moscow, 1979.

[2] Mikeš J., Vanvzurova A., Hinterleitner I. *Differential Geometry of Special Mappings.* Palacky Univ. Press, Olomouc, Czech Republic, second edition,2019.