2F-planar mappings of Riemannian spaces with a special affinor structure

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The concept of 2F-planar mapping ($2FPM$) of spaces with affine connection and Riemannian spaces was defined by R.J.Kadem[1]. These mappings are a natural generalization of geodesic [2] and F-planar mappings [3]. R.J.Kadem investigated general problems of $2FPM$ theory. In particular, he proved that every such mapping preserves affinor structure.

We study $2FPM$ of Riemannian spaces with a special type of f-structure

$$(V_n, g_{ij}, F_i^h) \longrightarrow (\overline{V}_n, \overline{g}_{ij}, F_i^h).$$

The fundamental equations of such a mapping in the common coordinate system (x^i) with respect to the $2FPM$ has the form:

$$\overline{\Gamma}_{ij}^h(x) = \Gamma_{ij}^h(x) + \psi_{(i}(x)\delta_{j)}^h + \phi_{(i}(x)F_{j)}^h(x) + \sigma_{(i}(x)F_{|\alpha|}^h(x)F_{j)}^\alpha(x), \tag{1}$$

$$F_i^h(x) = \overline{F}_i^h(x),$$

$$g_{i\alpha}F_j^\alpha = -g_{j\alpha}F_i^\alpha, \qquad \overline{g}_{i\alpha}F_j^\alpha = -\overline{g}_{j\alpha}F_i^\alpha, \tag{2}$$

$$F_{(i,j)}^h = q_{(i}F_{j)}^h, \tag{3}$$

$$F_\alpha^h F_\beta^\alpha F_i^\beta + F_i^h = 0, \tag{4}$$

$$i, h, j, ... = 1, 2, ...n,$$

where $\Gamma_{ij}^h, \overline{\Gamma}_{ij}^h$ are the Christoffel symbols of $V_n, \overline{V}_n$, respectively; $\psi_i(x)$, $\phi_i(x)$, $\sigma_i(x)$, $q_i(x)$ are certain covectors; $F_i^h(x)$ is affinor; brackets (i, j) denote the symmetrization with respect to the corresponding indices; comma «,» is a sign of the covariant derivative in respect to the connection of V_n.

We call an affinor structure F_i^h that satisfies conditions (3) a *generalized-recurrent structure* and $q_i(x)$ - the *generalized-recurrent vector*.

We have obtained the properties of the Riemannian and Ricci tensors of the generalized recurrent f-space.

The relationship between vectors $\psi_i(x)$, $\phi_i(x)$, $\sigma_i(x)$, $q_i(x)$ under conditions (1), (2), (3), (4) was found.

It is proved that the class of generalized recurrent spaces (V_n, g_{ij}, F_i^h) is closed with respect to $2FPM$, that is the space $(\overline{V}_n, \overline{g}_{ij}, F_i^h)$ under conditions (1), (2), (3), (4) is also generalized recurrent, but the vector of generalized recurrence is generally not preserved:

$$F_{(i|j)}^h = \tilde{q}_{(i}F_{j)}^h, \tag{5}$$

were «|» is a sign of the covariant derivative in respect to the connection of $\overline{V}_n$ and

$$\tilde{q}_i = q_i - \psi_i + \phi_\alpha F_i^\alpha + \sigma_i.$$

References

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[3] Mikeš J., Vanvzurova A., Hinterleitner I. *Differential Geometry of Special Mappings.* Palacky Univ. Press, Olomouc, Czech Republic, second edition,2019.