Integral problem for system of partial differential equations of higher order

Grzegorz Kuduk
(Faculty of Mathematics and Natural Sciences University of Rzeszow, Poland Graduate of University of Rzeszow, Poland)
E-mail: `gkuduk@onet.eu`

Let $H(\mathbb{R}_+ \times \mathbb{R}^n)$ be a class of entire functions on $\mathbb{R}$, K_L is a class of quasipolynomials of the form $\varphi(x) = \sum_{i=1}^n Q_r(x) \exp[\alpha_r x]$, where $\alpha_r \in L \subseteq \mathbb{C}$, $\alpha_k \neq \alpha_l$, for $k \neq l$, $Q_r(x)$ are given polynomials.

In the strip $\Omega = \{(t,x) \in \mathbb{R}^{n+1} : t \in \{(0,T), x \in \mathbb{R}^n\}$, we consider of the system of equations

$$\frac{\partial^n U_i}{\partial t^n} + \sum_{j=1}^{n} a_{ij}\left(\frac{\partial}{\partial x}\right)\frac{\partial^{n-j} U_i}{\partial t^{n-j}} = 0, \tag{1}$$

$$\int_0^T t^{n-j} U_i(t,x)dt = \varphi_{ik}(x), \quad k = \{1,...,n\}, \quad t \in [0,T], x \in \mathbb{R}^n. \tag{2}$$

Where $a_{ij}\left(\frac{\partial}{\partial x}\right)$, are differential expression with entire symbols $a_{ij}(\lambda) \neq 0$. Denote be P seet zeros of function $\eta(\lambda) = \int_0^T W^{n-1}(t,\lambda)dt$.

Theorem 1. ***Theorem.*** *Let $\varphi_{ik}(x) \in K_L$, $i = \{1,...,n\}$, $j = \{1,...,n\}$ then the class $K_{L\setminus P}$ exist and unique solution of the problem (1)-(2), can be represented in the form*

$$U_i(t,x) = \sum_{k=0}^{n-1}\sum_{p=1}^{n} \varphi_{kp}\left(\frac{\partial}{\partial x}\right)\left\{\frac{1}{\eta(\lambda)} T_{kjp}(t,\lambda) W(t,\lambda)\exp[\lambda x]\right\}\Bigg|_{\lambda=0},$$

Solution of the problem (1)-(2) according to the differential-symbol method [1,2],

References

[1] P.I. Kalenyuk, Z.N. Nytrebych. Generalized Scheme of Separation of Variables. Differential-Symbol Method. *Publishing House of Lviv Polytechnic Natyonaly University, 2002. – 292 p. (in Ukrainian).*

[2] G. Kuduk. Problem with integral conditions for system of partial differential equations of third order. *International Scientific Conference "Current problems of Mechanics and Mathematics - 2023" dedicated to 95th birth anniversary of Yaroslav Pidstryhach and 45th anniversary of the Pidstryhach Institute for Applied Problems of Mechanics and Mathematics. May 23 - 25, 2023, Lviv, Ukraine, p. 331-332.*