Chaotic properties of weighted shifts

Zoriana Novosad
(Lviv University of Trade and Economics)
E-mail: `zoriana.maths@gmail.com`

Let $T : X \mapsto X$ be a bounded linear operator acting on topological space X

Definition 1. The operator T is Li-Yorke chaotic if there is uncountable set $U \subset X$, called scrambled set, such that for each $x, y \in U$, $x \neq y$, $\lim\limits_{n \mapsto \infty} \|T^n(x) - T^n(y)\| = 0$

Let $(H_n)_{n=0}^{\infty}$ be a sequence of Hilbert spaces. Each space H_n is supposed to be nontrivial and possibly nonseparable.

Assume that for all n and m, the spaces H_n and H_m are isomorphic. We define $\ell_2(H_n) = \ell_2((H_n)_{n=0}^{\infty})$ as the Hilbert space consisting of elements $x = (x_0, x_1, \ldots, x_n, \ldots)$, $x_k \in H_k$ endowed with norm $\|x\| = \left(\sum_{i=0}^{\infty} \|x_i\|^2\right)^{\frac{1}{2}}$.

Let (ω_n) be a sequence of weights and let us fix a sequence of isomorphisms $J_m : H_m \to H_{m-1}$, $\|J_m\| = 1$, $m \in \mathbb{N}$

$$0 \longleftarrow H_0 \xleftarrow{J_1} H_1 \xleftarrow{J_2} \cdots \xleftarrow{J_n} H_n \cdots .$$

An operator

$$T \colon \ell_2(H_n) \to \ell_2(H_n)$$

will be called a *backward weighted shift (with respect to the family (J_m)) with weight sequence* (ω_n) if it is of the form

$$T(x) = (\omega_1 J_1(x_1), \omega_2 J_2(x_2), \ldots, \omega_m J_m(x_m), \ldots).$$

Theorem 2. *Let $(H_n)_{n=0}^{\infty}$ be a sequence of Hilbert spaces. A backward weighted shift $T \colon \ell_2(H_n) \to \ell_2(H_n)$, $T(x) = (\omega_1 J_1(x_1), \omega_2 J_2(x_2), \ldots, \omega_m J_m(x_m), \ldots)$ is Li-Yorke chaotic*

This research was supported by the National Research Foundation of Ukraine, 2023.03/0198 "Analysis of the spectra of countably generated algebras of symmetric polynomials and possible applications in quantum mechanics and computer science".

References

[1] T. Bermúdez, N.J. Kalton, *The range of operators on von Neumann algebras*, Proc. Amer. Math. Soc., 130: 1447-1455, 2002.

[2] Zoriana Novosad, Andriy Zagorodnyuk. *The Backward Shift and Two Infinite-Dimension Phenomena in Banach Spaces*, Symmetry, 15: P. 1855, 2023,