The structure of gradient flows with an internal saddle connection on the sphere with holes

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We consider all possible topological structures of typical one–parameter bifurcations of gradient flows on the 2-sphere with holes when number of singular points is at most six. Such flows were completely researched in [2, 3, 4].

Complete topological invariants can be established for the topological equivalence of flows. Usually it is a graph which is augmented with additional information. The purpose of the considered article is to construct a complete invariant for Morse flows and gradient codimension one gradient flows on the 2-sphere with holes which resembles a chord diagram for Morse flows, researched in [1]. This invariant has a marked point in the diagram which allows us to define clearly a number code of the flow. The invariants we have constructed (the distinguishing graph and the flow code) are generalizations of the distinguishing graph of Peixoto and the Oshemkov–Sharko code which had been developed for Morse flows on closed surfaces.

For example, we can consider gradient flows on 2-disk which have only one internal saddle connection whereas all their singular points are hyperbolic.

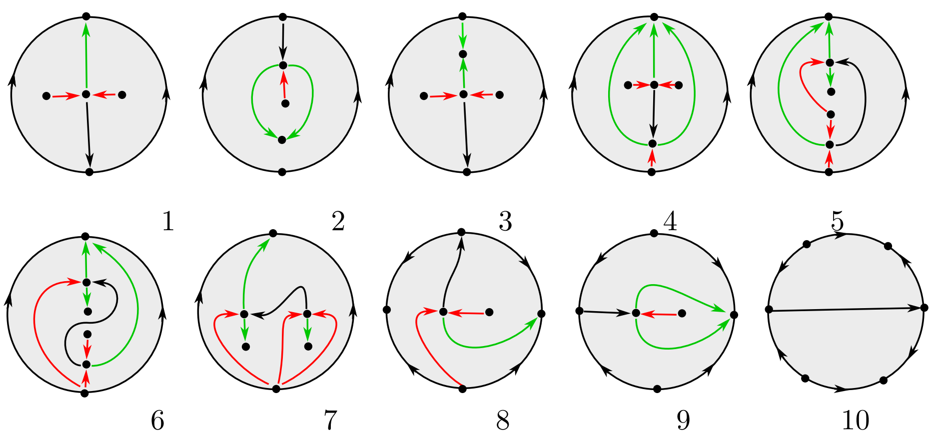

Figure 0.1. Saddle connections on D^2

All possible separatrix connections on a two-dimensional disk with no more than 6 singular points are depicted in Fig. 0.1. The diagrams of reverse flows can be obtained from these by changing all directions and colors of the separatrices (green and red swap places). With such a substitution, diagrams 6 and 10 will revert to themselves (they define one flow each). The remaining diagrams define two flows each.

References

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