Topological structure of pre-Hamiltonian flows on the projective plane

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Definition 1. *Pre-Hamiltonian flow* on the projective plane $\mathbb{R}P^2$ is a flow flows whose lift to the double cover (S^2) is a Hamiltonian flow on S^2 with a Hamiltonian that is a Morse function. A flow is *simple* if there are no saddle connections between different saddles within it. *Topological equivalence* of flows is a homeomorphism of the surface that maps trajectories to trajectories and preserve their direction.

For the topological classification of pre-Hamiltonian flows on $\mathbb{R}P^2$, we construct a complete topological invariant of the flow – a distinguishing graph. This invariant is a rooted oreinted tree and is a Reeb graph. In this case, Hamiltonian flows are divided into two types: 1) those whose root of the distinguishing graph has degree 1, and 2) those whose root has degree 2. All other vertices have degree 1 or 3.

Theorem 2. *Two simple prohamiltonian flows on the projective plane are topologically equivalent if and only if their distinguishing graphs are equivalent*

The presence of a marked vertex (root) in the distinguishing graph allows for the efficient computation of the number of topologically non-equivalent graphs with a given number of saddles.

Theorem 3. *The number of topologically non-equivalent simple pre-Hamiltonian flows with k saddles on the projective plane $\mathbb{R}P^2$ can be calculated using the formula*

$$N(\mathbb{R}P^2)_k = K_k + \sum_{i=0}^{k-1} K_i K_{k-i-1},$$

where

$$K_{2n} = 3(K_0K_{2n-1} + K_1K_{2n-2} + \ldots + K_{n-1}K_n),$$
$$K_{2n+1} = 3(K_0K_{2n} + K_1K_{2n-1} + \ldots + K_{n-1}K_{n+1}) + \frac{3K_n^2 + K_n}{2}.$$

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