Directional Maximal Operators and Kakeya-Type Sets

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A well-known result of the theory of differentiation of integrals is the *Lebesgue Differentiation Theorem*. This theorem states that for any integrable function $f \in L^1(\mathbb{R}^n)$, for almost every point $x \in \mathbb{R}^n$, the average value of $|f|$ over balls centered at x converges to $f(x)$ when the radius of these balls shrinks to zero. This important result is a consequence of the weak-type boundedness of the Hardy-Littlewood Maximal Operator in L^p spaces.

Naturally, one might ask whether this result remains true if we consider averages over other types of subsets, such as a collection of rectangles assigned to a set of directions.

In this talk, we will discuss a recent result that provides a condition on a set of directions $\Omega \subseteq \mathbb{S}^1$ sufficient to show the admissibility of Kakeya-type sets, extending prior work of Bateman and Katz. This condition guarantees that the associated directional maximal operator M_Ω is unbounded on $L^p(\mathbb{R}^2)$ for every $1 \leq p < \infty$.

References

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