On floating bodies and related topics

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This is a joint work with Maria Alfonseca, Fedor Nazarov, Alina Stancu and Vlad Yaskin.

Let K be a convex body in $\mathbb{R}^2$. For every $\theta \in \mathbb{R}$ and the corresponding unit vector $e(\theta) = (\cos\theta, \sin\theta)$ and for every $t \in \mathbb{R}$, define the half-planes

$$W^+(\theta, t) = \{x : \langle x, e(\theta)\rangle \geq t\} \quad \text{and} \quad W^-(\theta, t) = \{x : \langle x, e(\theta)\rangle \leq t\}.$$

If $0 < \mathcal{D} < 1$, then for every $\theta \in \mathbb{R}$, there is a unique $t(\theta)$ such that

$$\mathrm{vol}_2(W^+(\theta, t(\theta)) \cap K) = \mathcal{D}\,\mathrm{vol}_2(K).$$

The corresponding convex body of flotation $K^{\mathcal{D}}$ is defined as

$$K^{\mathcal{D}} = \bigcap_{\theta \in \mathbb{R}} W^-(\theta, t(\theta)).$$

We investigate the homothety conjecture for convex bodies of flotation of planar domains. We show that there is a density close to $\frac{1}{2}$ for which there is a body K different from an ellipse with the property that $K^{\mathcal{D}}$ is homothetic to K.

References

[1] , Two results on the homothety conjecture for convex bodies of flotation on the plane, Revista de la Unión Matemática Argentina (2025).