On fuzzy K-ultrametric spaces

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Let $K \in [0, \infty]$. A metric space (X, d) is called a K-ultrametric space [1, 2] if $d(x, y) \leq K$ whenever $\min\{d(x, z), d(z, y)\} \leq K$. The talk is devoted to a counterpart of this notion in the realm of fuzzy metric spaces [3].

A binary operation $*\colon [0, 1] \times [0, 1] \to [0, 1]$ is a continuous t-norm if it satisfies the following conditions.

(1) $*$ is associative and commutative,

(2) $*$ is continuous,

(3) $a * 1 = a$ for all $a \in [0, 1]$,

(4) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$, for each $a, b, c, d \in [0, 1]$.

A triple $(X, M, *)$ is called a fuzzy metric space if X is a nonempty set, $*$ is a continuous t-norm and $M\colon X \times X \times (0, \infty) \to \mathbb{R}$ is a map such that for every $x, y, z \in X$ and $t, s > 0$ we have

1) $0 < M(x, y, t) \leq 1$;

2) $M(x, y, t) = 1$ if and only if $x = y$;

3) $M(x, y, t) = M(y, x, t)$;

4) $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$;

5) $M(x, y, \cdot)\colon (0, \infty) \to (0, 1]$ is continuous.

A triple $(X, M, *)$ is called a fuzzy ultrametric space if X is a nonempty set, $*$ is the minimum $\wedge$ and $M\colon X \times X \times (0, \infty) \to \mathbb{R}$ is a map satisfying conditions 1), 2), 3) and 5) of this definition and moreover

4') $M(x, y, t) * M(y, z, s) \leq M(x, z, \max\{t, s\}$ for $x, y, z \in X$ and $t > 0$. Condition 4') is equivalent to the condition $M(x, y, t) * M(y, z, t) \leq M(x, z, t)$.

Given a nondecreasing function $K\colon (0, \infty) \to [0, 1]$, we define a fuzzy K-ultrametric as a fuzzy metric satisfying the condition $M(x, y, t) \wedge M(y, z, t) \leq M(x, z, t)$ whenever $\max\{M(x, z, t), M(z, y, t)\} \geq K(t)$.

We establish some properties of fuzzy K-ultrametrics and consider questions of fuzzy K-ultrametrization of products, hyperspaces, and spaces of measures on fuzzy K-ultrametric spaces.

References

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