On ruled affine submanifolds with two-dimensional base

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When we study affine submanifolds with flat connection [1, 2, 3] or rank two affine fundamental form [4], it often turns out that representatives of such submanifolds are ruled submanifolds. So, they deserve detailed research.

We consider generalized ruled affine submanifolds, namely ruled affine submanifolds with 2-dimensional base and $n-2$ rulings, in case of codimension 1 and 2. We obtain such a type of ruled affine submanifolds (codimension 2) when we study affine submanifolds of rank two [4].

Detailed description of ruled affine submanifolds of arbitrary dimension and codimension in the classical sense, that is, ruled submanifolds over a curve, can be found in [5].

In case of codimension 1 the base of the ruled affine submanifold is a hyperbolic-type surface in $\mathbb{R}^3$. In case of codimension 2 the base of the ruled affine submanifold is an elliptic-type surface in $\mathbb{R}^4$. We find conditions on the directions of rulings that follow from the condition of completeness of the affine immersion. In particular, we receive all affine characteristics (induced connection, transversal connection, affine fundamental forms, Weingarten operators, curvature tensor) of such an affine immersion in case the base surface is a complex curve.

References

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