Categorical Version of Shilov's Theorem on Closed Equivalence Relations

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Let X be a compact Hausdorff topological space, $A = C(X)$, and let B be a closed self-adjoint subalgebra of the algebra A. Then the classic Shilov's theorem says that equivalence relation R_B on the space X, defined by the formula:

$$R_B = \{(x, y) \in X \times X : \forall b \in B,\ b(x) = b(y)\}$$

is a closed equivalence relation, and B is the algebra of functions invariant under R_B.

This work explores the relationship between the category of closed equivalence relations on compact topological spaces and the category of pairs of commutative C^*-algebras. It is shown that these categories are equivalent.

To describe this equivalence, the functors C and Σ are constructed. The functor C assigns to an object (X, R) in the category of closed equivalence relations the pair $(C(X), B_R)$, where $C(X)$ is the algebra of continuous functions on X and B_R is the algebra of invariant functions with respect to R. The functor Σ assigns to an object (A, B) in the category of pairs of commutative C^*-algebras the spectrum of this pair, that is spectrum $\Sigma(A)$ of the algebra A and a natural equivalence relation on $\Sigma(A)$.

References

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