Fejer's method in extremal problems of geometric complex analysis

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For the class of typically real in the unit disc of the complex plane polynomials, the results of W. Rogosinski and G. Szegö [1] implies the sharp estimates for the second coefficient, however the problem of finding the extremizers still open.

Within algebraic framework, we construct explicit polynomials which attain these bounds and prove their uniqueness. The proof uses the Fejér - Riesz representation of nonnegative trigonometric polynomials, a 7-band Toeplitz matrix of arbitrary finite dimension, and Chebyshev polynomials of the second kind and their derivatives.

References

[1] Rogosinski, W. W. and Szegö, G., Extremum problems for non-negative sine polynomials, *Acta Sci. Math. (Szeged)*, 12:112–124, 1950.