On n-convex $(2n-2)$-dimensional submanifolds in E^{2n}

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Let us recall the following definition (see [2, §16]).

Definition 1. A subset C of the Euclidean space E^n is called k-convex if through each point $x \in E^n \setminus C$ there passes a k-dimensional plane π_x that does not intersect C.

The following theorem is an n-dimensional generalization of our recent result obtained in [1].

Theorem 2. *If $S \subset E^{2n}$ is an n-convex closed $(2n-2)$-dimensional submanifold which is C^2-embedded into E^{2n}, then it admits a mapping of degree one to $S^{n-1} \times S^{n-1}$. If S is non-orientable, we assume that the degree is taken modulo 2.*

Corollary 3. *Neither S^{2n-2} nor $S^k \times S^{2n-k-2}$, $1 \le k \le n-2$, admits a C^2-embedding into E^{2n} as an n-convex submanifold.*

References

[1] Dmitry V. Bolotov. On 2-convex non-orientable surfaces in four-dimensional Euclidean space. *Journal of Mathematical Physics, Analysis, Geometry*, 21(4):371–379, 2025. doi:10.48550/arXiv.2412.19757.

[2] Yuri B. Zelinsky. *Convexity. Selected Chapters*, volume 92 of *Proceedings of the Institute of Mathematics of the National Academy of Sciences of Ukraine*. Institute of Mathematics, NAS of Ukraine, Kiev, 2012.