Quadric surfaces of revolution as cubic Weingarten surfaces

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While the geometry of quadric surfaces of revolution is well-established within Euclidean space, their behavior and classification within the three-dimensional sphere $\mathbb{S}^3$ have remained largely under-examined. This presentation addresses this theoretical oversight by introducing a systematic taxonomy of non-degenerate spherical quadrics—specifically ellipsoids, hyperboloids, and paraboloids. These are analyzed as surfaces generated by the revolution of spherical conics around geodesic axes that are either incident to their foci or oriented orthogonally to them.

By utilizing spherical angular momentum as a primary geometric invariant, this study identifies these quadrics as a distinguished category of Weingarten surfaces (cf. [4]). We demonstrate that these surfaces are uniquely determined by a specific cubic functional relation between their principal curvatures. Furthermore, these results reveal an unexpected structural consistency across different ambient spaces; the cubic Weingarten relations identified in $\mathbb{S}^3$ directly parallel those previously documented in both Euclidean and Lorentzian frameworks (see [1], [2] and [3]). This suggests a fundamental universality in the curvature dynamics of quadric surfaces regardless of the underlying ambient manifold's signature or curvature.

References

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