On p-adic Differential Operators

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The p-adic differential operator, an important concept in functional and numerical analysis, plays a prominent role in the study of dynamical systems and differential equations in the non-Archimedean sense. Calculating the index of a differential operator shows the number of independent solutions of a differential equation. This helps to better understand the operator, particularly if there is a singularity present. This study deals with the measurement of irregularity in a disk. The effect of regular and irregular singularities on the index of the operator is analyzed. The calculation of the index, including Adolphson's method [1], is considered, and new, more precise and adaptable analytical techniques are proposed. In summary, our work aims to bridge local and global perspectives in p-adic differential equations, providing a framework for future research in this area. The interplay between local and global indices is crucial for understanding the structure of solutions in p-adic differential equations [3, 2].

Through a differential operator L, we study a "mod p differential system" $A(y) = 0$ on a disk D that is over a p-adic disk D^*. In this talk, we present the evaluation of the index of such a differential system. Also, we will measure the irregularity of such p-adic differential systems in a disk. We also proposes a method to connect local and global indices using the Mittag-Leffler theorem. It aims to enhance our understanding of the interplay between local solutions and global behavior in p-adic differential equations. These equations behave very differently from classical ones. This is mainly due to the non-Archimedean nature of the field, which affects convergence and the behavior of solutions. As a result, special methods are required, especially when studying singular points and computing the index of operators. This study highlights the importance of understanding the unique properties of p-adic differential equations, particularly in relation to their singular points and irregularities in solutions.

References

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