On Carathéodory boundary extension of mappings with moduli inequality

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All definitions and notions used below may be found in [1]. Let $A = A(x_0, r_1, r_2) = \{x \in \mathbb{R}^n : r_1 < |x - x_0| < r_2\}$, let $B(x_0, r_0) = \{x \in \mathbb{R}^n : |x - x_0| < r_0\}$, let $M_p(\cdot)$ be the modulus of family of paths of the order $p \geqslant 1$, and let $dm(y)$ be an element of the Lebesgue measure in $\mathbb{R}^n$, $n \geqslant 2$. Given sets E, $F \subset \overline{\mathbb{R}^n}$ and a domain $D \subset \mathbb{R}^n$ we denote by $\Gamma(E, F, D)$ a family of all paths $\gamma : [a, b] \to \overline{\mathbb{R}^n}$ such that $\gamma(a) \in E, \gamma(b) \in F$ and $\gamma(t) \in D$ for $t \in (a, b)$. If $f : D \to \mathbb{R}^n$, $y_0 \in f(D)$ and $0 < r_1 < r_2 < d_0 = \sup\limits_{y \in f(D)} |y - y_0|$, then by $\Gamma_f(y_0, r_1, r_2)$ we denote the family of all paths γ in D such that $f(\gamma) \in \Gamma(S(y_0, r_1), S(y_0, r_2), A(y_0, r_1, r_2))$. Let $Q : \mathbb{R}^n \to [0, \infty]$ be a Lebesgue measurable function. We say that f *satisfies Poletsky inverse inequality* at the point $y_0 \in \overline{f(D)}$, if the relation

$$M(\Gamma_f(y_0, r_1, r_2)) \leqslant \int\limits_{A(y_0,r_1,r_2)\cap f(D)} Q(y) \cdot \eta^n(|y - y_0|)\, dm(y) \qquad (1)$$

holds for any Lebesgue measurable function $\eta : (r_1, r_2) \to [0, \infty]$ such that $\int\limits_{r_1}^{r_2} \eta(r)\, dr \geqslant 1$.

Theorem 1. *Let D and D' be domains in $\mathbb{R}^n$, $n \geqslant 2$, and let D be a domain with a weakly flat boundary. Suppose that f is open discrete mapping of D onto D' satisfying the relation (1) at each point $y_0 \in \overline{D'}$. In addition, assume that the following conditions are fulfilled:*

1) for each point $y_0 \in \partial D' \setminus \{\infty\}$ there is $0 < r_0 \leqslant \sup\limits_{y \in D'} |y - y_0|$ such that, for any $0 < r_1 < r_2 < r_0$ there exists a set $E_1 \subset [r_1, r_2]$ of positive linear Lebesgue measure such that Q is integrable on $S(y_0, r)$ for $r \in E_1$;

2) $C(f, \partial D) \subset E$ for some set $E \subset \overline{D'}$ which is closed in $\overline{\mathbb{R}^n}$, while D' is locally finitely connected with respect to E, in other words, for each point $z_0 \in E$ and for any neighborhood U of this point there exists a neighborhood $V \subset U$ of z_0 such that the set $V \cap (D' \setminus E)$ consists of a finite number of components;

3) the set $f^{-1}(E \cap D')$ is nowhere dense in D.

Then the mapping f has a continuous extension $\overline{f} : \overline{D} \to \overline{D'}$, moreover, $\overline{f}(\overline{D}) = \overline{D'}$.

This result is published in [2].

References

[1] Martio O., Ryazanov V., Srebro U. and Yakubov E. *Moduli in Modern Mapping Theory.* Springer Science + Business Media, LLC : New York, 2009.

[2] Desyatka V., Sevost'yanov E. *Carathéodory boundary extensions for generalized quasiregular mappings.* https://ems.press/journals/zaa/articles/14299657.

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