Algebraic and Geometric Aspects of Non-Classical Knots

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Classical knot theory is built on the principle that crossings in a diagram can be freely manipulated via Reidemeister moves. In recent years, several extensions of this framework have emerged by modifying the behavior of crossings, leading to new classes of knot-like objects. In this talk, we survey a range of such non-classical knot theories, including *pseudo knots* [6], *singular knots* [4] and *bonded knots* [2], emphasizing their connections with braid theory and algebraic structures [3, 5].

We then introduce the notion of *stuck knots* [1], where certain crossings are rigid and cannot be altered by Reidemeister moves. This rigidity leads to new phenomena, including modified equivalence relations, novel invariants and the concept of unsticking distance. We discuss how classical tools, such as skein relations and HOMFLYPT-type invariants, extend to these settings, highlighting the robustness of algebraic constructions across different crossing behaviors.

Finally, we outline a program toward a theory of stuck braids, aiming to establish analogues of Alexander and Markov theorems and to develop associated algebraic frameworks. This perspective suggests a unifying approach to studying knot theory through the lens of crossing behavior, bridging geometry, algebra and emerging computational directions.

References

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