A stochastic Poisson coalgebra method and its applications

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This talk provides a practical approach to stochastic Lie systems, i.e. stochastic differential equations whose general solutions can be written as a function depending only on a generic family of particular solutions and some constants related to initial conditions [1, 2]. We correct the stochastic Lie theorem characterising stochastic Lie systems, which retains its classical form in the Stratonovich approach. New generalisations of stochastic Lie systems, like the so-called stochastic foliated Lie systems, are introduced. Subsequently, I focus on stochastic Lie systems that are Hamiltonian systems relative to different geometric structures [5]. Special attention is paid to the symplectic case. I study their stability properties and lay the foundations of a stochastic energy-momentum method. A stochastic Poisson coalgebra method is developed to derive superposition rules for Hamiltonian stochastic Lie systems [3, 4]. The present results complement previous approaches through the use of stochastic differential equations, rather than deterministic equations intended to reproduce certain features of stochastic models.

References

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